Investigation

Palindromes

You will need a hundred chart and coloured pencils.

A **palindrome** is a word, a phrase, or a number that reads the same from both directions.
Here are some examples of palindromes:

- mom
- never odd or even
- level
- 3663

Many numbers, such as 7, 11, and 232, are palindromes.
If a number is not a palindrome, follow these steps to make it a palindrome:

Reverse the digits.	67
Add the reverse number to the original number.	+ 76 143
Continue to reverse and add until the sum is a palindrome.	143 + 341 484

If you follow these steps, all the numbers from 1 to 100 will eventually become palindromes.

Part 1

➤ Use a hundred chart.
Shade the numbers that are palindromes yellow.
For the numbers that are not palindromes, reverse the digits and add to make palindromes.
Shade the numbers that become palindromes:
– in 1 step blue
– in 2 steps orange
– in 3 steps green
– in 4 steps red
– in more than 4 steps purple

➤ How are the numbers that became palindromes in 1 step related?
In 2 steps? In 3 steps? In 4 steps?
Describe any patterns you found.

Part 2

➤ A decimal such as 63.36 is a palindrome.
Why is a decimal such as 8.48 not a palindrome?

➤ Use the method from Part 1 to make palindrome decimals from these decimals.
7.1 6.5 4.7 3.65 4.81
How do the results for 6.5 and 4.7 compare to the results for 65 and 47?

Display Your Work

Create a summary of your work.
Use pictures, numbers, and words.

Take It Further

The years 1991 and 2002 are palindromes.
They are 11 years apart.
What is the next pair of palindrome years that are 11 years apart? What was the previous pair?
How far apart are palindrome years usually?

UNIT 1

Patterns and

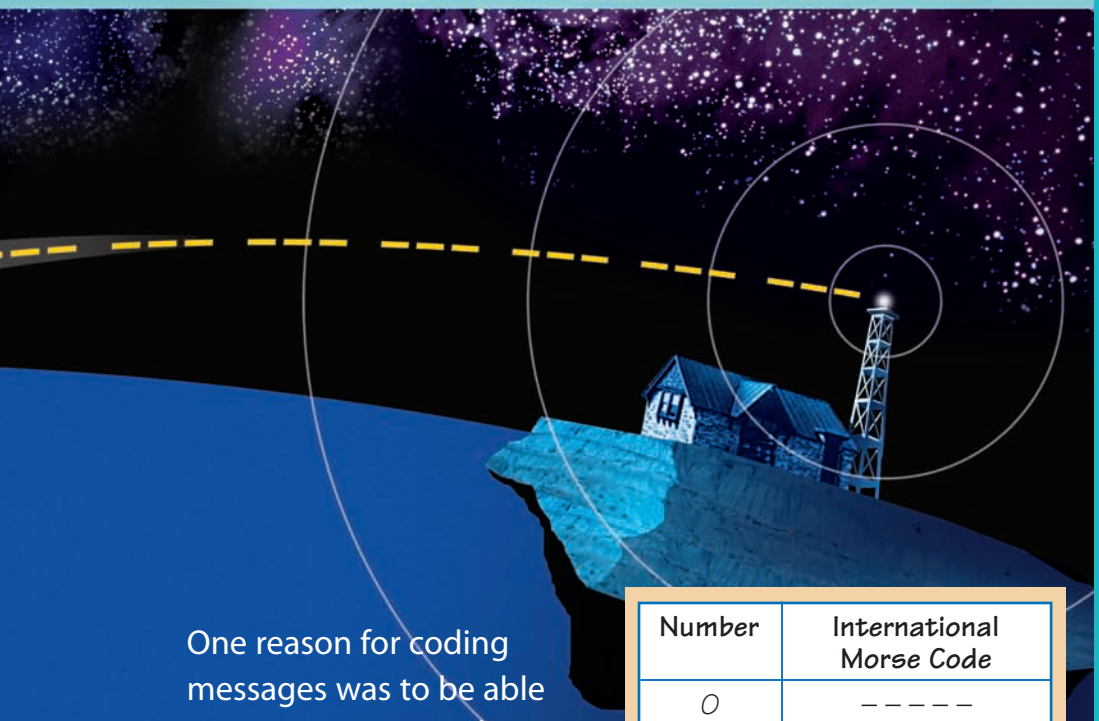
Crack the Code!

Guglielmo Marconi received the first transatlantic wireless communication on December 12, 1901. Morse code for the letter "s" was sent from Poldhu, Cornwall, England to Signal Hill, St. John's, Newfoundland.

Learning Goals

- describe patterns and relationships using graphs and tables
- use equations to represent number relationships
- use relationships within tables of values to solve problems
- identify and plot points in a Cartesian plane
- demonstrate the preservation of equality

Equations



One reason for coding messages was to be able to communicate without using a spoken language.

Morse code was developed by Samuel Morse almost 175 years ago.
It uses dots and dashes to represent letters, numbers, and punctuation.

Number	International Morse Code
0	– – – – –
1	• – – – –
2	• • – – –
3	• • • – –
4	• • • • –
5	• • • • •
6	– • • • •
7	– – • • •
8	– – – • •
9	– – – – •

- What other reasons might there be for coding a message?
- What patterns do you see in the Morse code for numbers?
- How would you write the number 503 in Morse code?

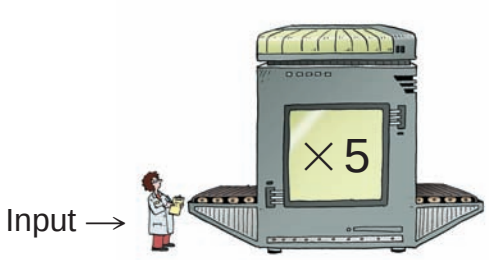
Key Words

- Input/Output machine
- coordinate grid
- Cartesian plane
- origin
- coordinates
- ordered pair
- horizontal axis
- vertical axis
- commutative property of addition
- commutative property of multiplication
- preservation of equality
- equivalent form of an equation

LESSON 1

Input/Output Machines

Look at this **Input/Output machine**.
Any number that is put into this machine is multiplied by 5.
When you input 6, the output is 30.
Suppose you input 9.
What will the output be?



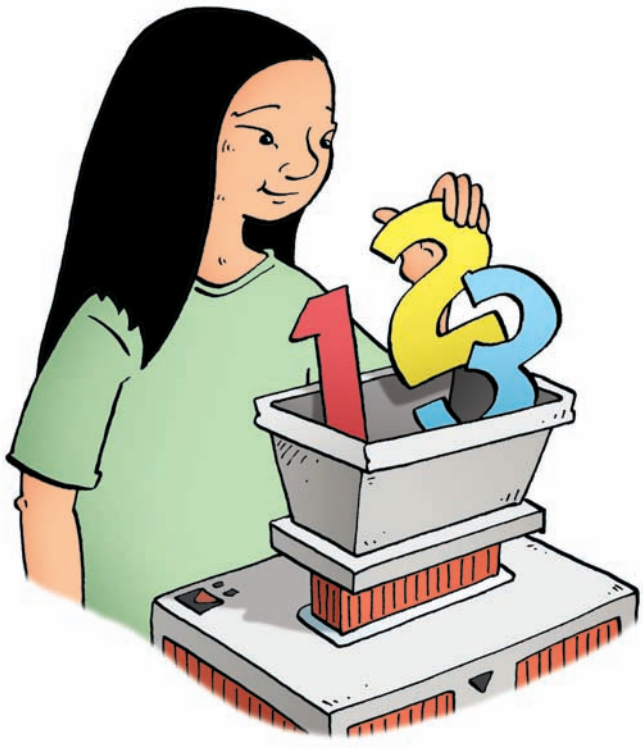
Explore

➤ Draw your own Input/Output machine.
Choose a number to go inside your machine.
Choose an operation.
Use your machine to create a number pattern.

An *operation* is add, subtract, multiply, or divide.

➤ Copy and complete this table of values for your pattern.
Write the pattern rule for the output numbers.

Input	Output
1	
2	
3	



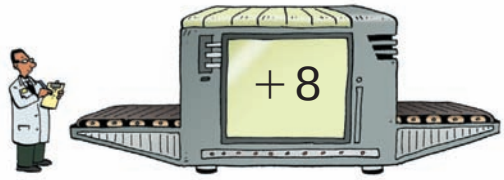
Show *and* Share

Share your machine and table of values with another pair of classmates.
Use your classmates' machine to extend their number pattern.

Lesson Focus | Explore the pattern within each column of a table of values.

Connect

We can use an Input/Output machine to make a growing pattern.



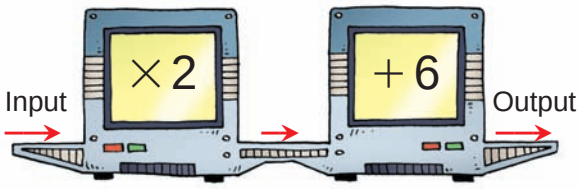
➤ This machine adds 8 to each input to get the output.

The pattern rule that relates the input to the output is: Add 8 to the input.

Input	Output
1	9
2	10
3	11
4	12

When each input increases by 1, the output increases by 1.
The pattern rule for the input is:
Start at 1. Add 1 each time.
The pattern rule for the output is:
Start at 9. Add 1 each time.

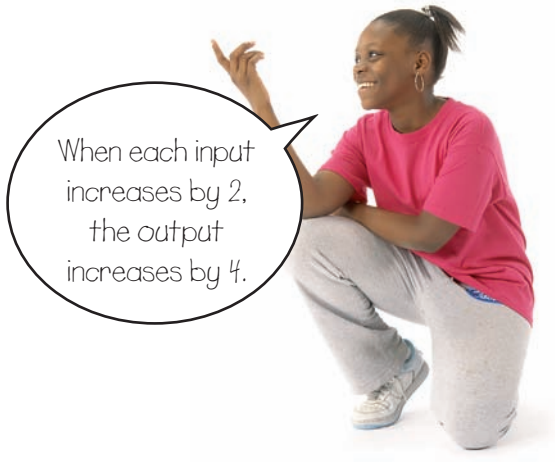
➤ This Input/Output machine doubles each input, then adds 6.



The pattern rule that relates the input to the output is:
Multiply the input by 2, then add 6.

Input	Output
2	10
4	14
6	18
8	22

The pattern rule for the input is:
Start at 2. Add 2 each time.
The pattern rule for the output is:
Start at 10. Add 4 each time.



Practice

1. For each Input/Output machine:

- Copy and complete the table.
- Write the pattern rule that relates the input to the output.
- Write the pattern rule for the input.
- Write the pattern rule for the output.

Input	Output
1	
2	
3	
4	
5	

a)

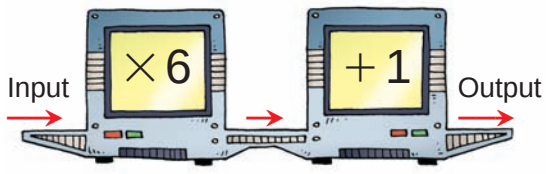
b)

2. For each Input/Output machine:

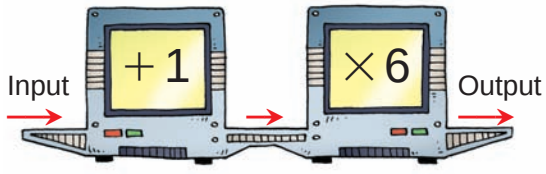
- Copy and complete the table.
- Write the pattern rule that relates the input to the output.
- Write the pattern rule for the input.
- Write the pattern rule for the output.

Input	Output
2	
4	
6	
8	
10	

a)



b)



3. Look at question 2 and your tables.

a) How are the Input/Output machines the same? How are they different?

b) How do the output numbers from the two machines compare? Explain.

c) Is it possible to get more than one output number for each input? How do you know?

4. Copy and complete this table.
The pattern rule that relates the input to the output is:
Divide the input by 6.

a) Write the pattern rule for the input.

b) Write the pattern rule for the output.

Input	Output
36	
42	
48	
54	
60	

5. Copy and complete this table.
The pattern rule that relates the input to the output is:
Divide the input by 3, then subtract 2.

a) Write the pattern rule for the input.

b) Write the pattern rule for the output.

Input	Output
30	
60	
90	
120	
150	

6. The pattern rule that relates the input to the output is:
Add 4 to the input. Then divide by 2.
Check the data in the Input/Output table.
Identify any output numbers that are incorrect.
How do you know they are incorrect?
Show your work.

Input	Output
4	2
8	4
16	10
26	15
30	19

7. The pattern rule that relates the input to the output is:
Divide the input by 6, then add 5.

a) Check the data in the Input/Output table. Identify any output numbers that are incorrect. How do you know they are incorrect?

b) Correct the table.

c) Write 3 more input and output numbers for this pattern rule.

Show your work.

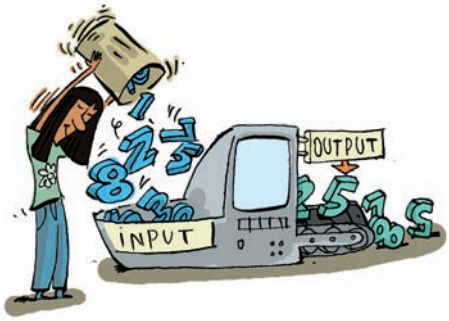
Input	Output
6	6
12	7
30	10
42	2
54	15

8. The pattern rule that relates the input to the output is:
Multiply the input by 4. Then subtract 3.
Find the missing numbers in the table.
How can you check your answers?

Input	Output
3	9
6	?
9	?
12	45
15	?

9. The pattern rule that relates the input to the output is:
Add 5 to the input. Then multiply by 3.
Find the missing numbers in the table.
What strategies did you use?

Input	Output
2	21
5	?
?	39
11	?
?	57
?	66



10. **a)** Draw an Input/Output machine with two operations.
Choose two numbers and two operations for your machine.
b) Choose 5 input numbers.
Find the output numbers.
c) Erase 2 input numbers and 2 output numbers.
Each row must have at least one number.
Trade tables with a classmate.
Trade pattern rules that relate the input to the output.
Find your classmate's missing numbers.

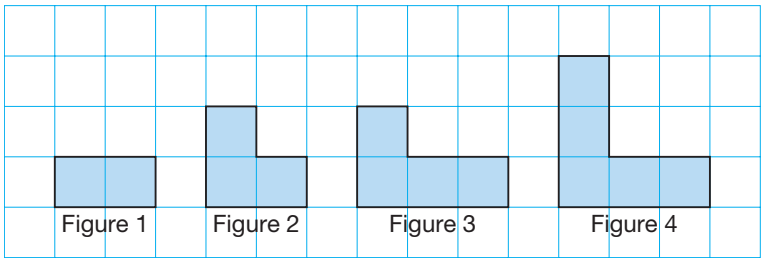
Reflect

Suppose you want to make an Input/Output machine to convert millimetres to metres.
Describe what your machine would look like.

LESSON 2

Patterns from Tables

How does this pattern of squares represent the table of values?



Input	Output
1	2
2	3
3	4
4	5

Explore

You will need toothpicks and dot paper.

- Build 5 figures to represent the pattern in this table. Make sure the figures show a pattern.
- Draw each figure in the pattern on dot paper.
- What patterns do you see in the figures? In the table?
- Write a pattern rule that relates each figure number to the number of toothpicks. Predict the number of toothpicks needed to build the 7th figure. Use toothpicks to check.

Figure	Number of Toothpicks
1	3
2	5
3	7
4	9
5	11

Show *and* Share

Compare your patterns and drawings with those of another pair of classmates.
Are your drawings the same or different?
If they are different, do both sets of drawings represent the table of values? Explain.
What Input/Output machine could you use to represent the table?

Connect

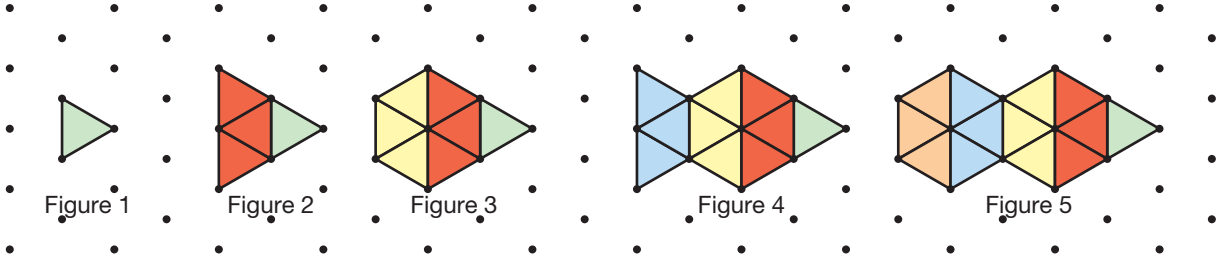
➤ We can draw pictures to show the relationship in a table of values.

Input	Output
1	1
2	4
3	7
4	10
5	13

3
3
3
3

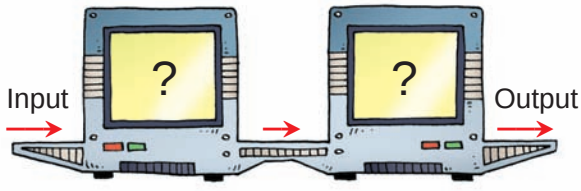
In this table:
The input increases by 1 each time.
The output increases by 3 each time.

We could draw a pattern of triangles on triangular dot paper.
The figure number is the input.
The number of triangles in each figure is the output.



➤ We can use a pattern rule to describe the relationship between the 2 columns in a table of values.
This pattern rule tells us the numbers and operations in the corresponding Input/Output machine.

The table shows the input and output for this two-operation machine.



Input	Output
1	1
2	5
3	9
4	13
5	17

4
4
4
4

To identify the numbers and operations in the machine:

Think:

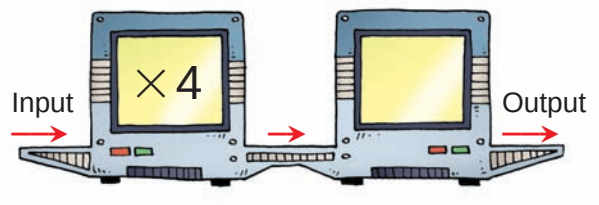
When the output increases by 4, that is a clue about what to do.

The pattern rule for the output is:

Start at 1. Add 4 each time.

This suggests that the input numbers are multiplied by 4.

The output increases by 4. Each input must be multiplied by 4.

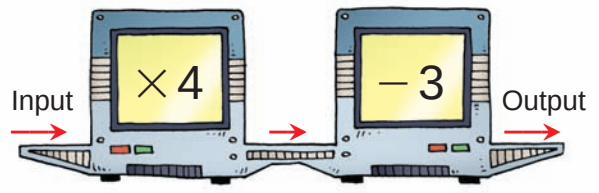


Look at the input 2.
Multiply by 4.
$2 \times 4 = 8$
But, the output is 5.

I check all the inputs to make sure I have found the correct numbers and the correct operations.

Think:

I have 8. To get 5, I subtract 3.
So, -3 goes into the second part of the machine.
$8 - 3 = 5$



This Input/Output machine multiplies each input by 4, then subtracts 3.
The pattern rule that relates the input to the output is:
Multiply the input by 4.
Then subtract 3.

We can use this rule to predict the output for any input.
For an input of 8, the output should be:
$8 \times 4 - 3 = 29$

We can check this by extending the table.
Add 1 to each input and 4 to each output.

Input	Output	
1	1	
2	5	4
3	9	4
4	13	4
5	17	4
6	21	4
7	25	4
8	29	4

Practice

1. Each table shows the input and output from a machine with one operation. For each table:
   - Identify the number and the operation in the machine.
   - Continue the patterns.
     Write the next 4 input and output numbers.
   - Write the pattern rule that relates the input to the output.

a)

Input	Output
1	7
2	14
3	21
4	28

b)

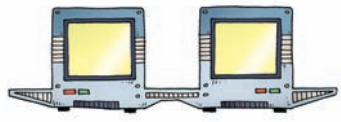
Input	Output
50	39
49	38
48	37
47	36

c)

Input	Output
2	20
4	40
6	60
8	80

d)

Input	Output
500	485
450	435
400	385
350	335



2. Each table shows the input and output from a machine with two operations. For each table:
   - Identify the numbers and the operations in the machine.
   - Choose 4 different input numbers. Find the output for each input.
   - Predict the output when the input is 10. Check your prediction.

a)

Input	Output
1	2
2	5
3	8
4	11

b)

Input	Output
1	9
2	14
3	19
4	24

c)

Input	Output
3	3
4	5
5	7
6	9

d)

Input	Output
4	17
5	21
6	25
7	29

3. Use the table of values in question 2a.
Draw pictures to show the relationship in the table.

4. Each table shows the input and output from a machine with two operations.
   - Find the pattern rule that relates the input to the output.
   - Use the pattern rule to find the missing numbers in the table.
   - Use the patterns in the columns to check your answers.
   - Predict the output when the input is 40. Check your prediction.

a)

Input	Output
5	21
6	24
7	27
?	30
9	?
10	?

b)

Input	Output
0	1
5	2
10	3
?	4
20	?
25	?

5. You may need Colour Tiles or counters, and dot paper.
   a) Use tiles, counters, or pictures to show the relationship in this table. Record your work.
   b) Write a pattern rule that relates the input to the output.
   c) Predict the output when the input is 9.
   Extend your pictures to check.
   d) Which input has an output of 28?
   Describe the strategy you used to find out.

Input	Output
1	6
2	8
3	10
4	12

6. **a)** Draw an Input/Output machine with two operations.
Choose two numbers and two operations for your machine.
   b) Choose 5 input numbers. Find the output numbers.
   c) Trade tables with a classmate.
   Find the pattern rule that relates the input to the output.
   Use this pattern to write the next 4 input and output numbers.

Reflect

When you look at an Input/Output table, what strategies do you use to identify the numbers and operations in the machine?

LESSON 3

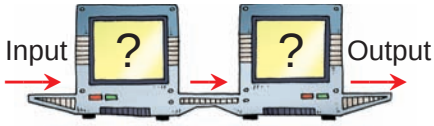
Strategies Toolkit

Explore

Abi made an Input/Output machine that uses two operations.

Here is a table for Abi's machine.

Find out what the machine does to each input number.



Input	Output
15	6
5	4
20	7
25	8
10	5

Show *and* Share

Explain the strategy you used to solve the problem.

Connect

Ben made an Input/Output machine that uses two operations. Here is a table for Ben's machine. What does Ben's machine do to each input number?

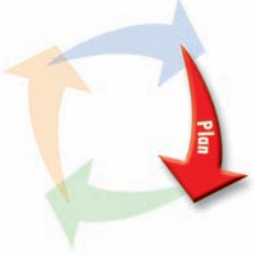
Input	Output
2	13
4	23
6	33
8	43
10	53

Strategies

- Make a table.
- Solve a simpler problem.
- Guess and test.
- Make an organized list.
- Use a pattern.

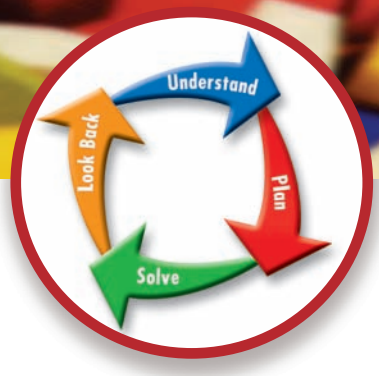
What do you know?

- The machine uses two operations on an input number.



Think of a strategy to help you solve the problem.

- You can **use a pattern**.
- Analyse the pattern in the *Output* column to find out what the machine does to each input number.



The output numbers increase by 10.
This suggests the input numbers are multiplied by 10. Look at input 2.
Multiply by 10: $2 \times 10 = 20$
But the output is 13.
We subtract 7 from 20 to get 13.

Check: Look at input 4.
Multiply by 10: $4 \times 10 = 40$
Subtract 7: $40 - 7 = 33$
The output should be 23.
This pattern rule does not work.

Try a different pattern.
When the input increases by 2,
the output increases by 10.
So, when the input increases by 1,
the output increases by $10 \div 2 = 5$.
This suggests the pattern involves multiples of 5.
Which two operations does Ben's machine use?

Input	Output
2	13
3	18
4	23
5	28
6	33

Use the operations in the machine to extend the pattern of the output numbers.
Check that the rule is correct.

Practice

Choose one of the **Strategies**

1. Design an Input/Output machine for each table below.
How did you decide which operations to use?

a)

Input	Output
2	7
4	15
6	23
8	31

b)

Input	Output
3	10
6	19
9	28
12	37

Reflect

Choose one part of question 1.
Explain how you used a pattern to solve it.

What's My Rule?

You will need a set of 10 blank cards for each player.
The object of the game is to be the first player to guess another player's rule.

Before the game begins, each player should:

- Label one side of each card "Input" and the other side "Output."
  Label the Input side of each card with the numbers 1 to 10.
- Choose a secret rule. You can use one or two operations.
  Write your rule on a separate piece of paper.
- Apply your rule to the number on the Input side of each card.
  Write the resulting number on the Output side of that card.
- Shuffle your cards. Place them in a pile.

To play:

- Player 1 shows all players both sides of her top card.
  Players record the input and output numbers in a table of values.
- Player 1 continues to show both sides, one card at a time.
  After each card is shown, Player 1 asks if anyone can guess the rule.
  The player who guesses the rule gets 1 point.
  A player who guessed incorrectly cannot guess again
  until every other player has had a guess.
  If no one guesses the rule after all 10 cards have been shown, Player 1 gets 1 point.
- Player 2 has a turn.
  Play continues until all players have shown their cards.

LESSON 4

Using Variables to Describe Patterns

Which expression below represents this number pattern?
34, 35, 36, 37, 38, . . .
$33 + t$ $33 - t$ $34 + t$

Explore

A Grade 6 class plans to go to the Winnipeg Planetarium.
The cost to rent the school bus is $75.
The cost of admission is $5 per student.

➤ Make a table of values to show the total cost for 1, 2, 3, 4, 5, and 6 students.

Number of Students	Total Cost ($)
1	
2	

➤ What patterns do you see in the table?
Write a pattern rule that relates the number of students to the total cost.

➤ Use the pattern rule to find the cost for 25 students.

➤ Suppose the total cost was $180.
How many students would be on the trip?
How did you find out?

Show *and* Share

Share your pattern rule and answers with another pair of classmates.
How did the patterns in the table help you solve the problem?
If your pattern rules are the same, work together to use a variable to write an expression to represent the pattern.

Connect

➤ To find the pattern rule that relates the input to the output:

Input	Output
1	7
2	11
3	15
4	19
5	23

The pattern rule for the output is:
Start at 7. Add 4 each time.
This suggests the input numbers are multiplied by 4.
Look at input 2.
Multiply by 4: $2 \times 4 = 8$
To get output 11, add 3.

The pattern rule that relates the input to the output is:
Multiply the input by 4. Then add 3.
We can use a variable in an expression to represent this rule.
Let the letter n represent any input number.

Input	Output
1	$4 \times 1 + 3 = 7$
2	$4 \times 2 + 3 = 11$
3	$4 \times 3 + 3 = 15$
4	$4 \times 4 + 3 = 19$
5	$4 \times 5 + 3 = 23$
⋮	⋮
n	$4 \times n + 3$

Then, the expression $4n + 3$ relates the input to the output.

$4n$ is the same as $4 \times n$.

➤ We can use a pattern to solve a problem.

Minowa works at a fishing camp in the Yukon.
Minowa earns $25 a day, plus $8 for each fishing net she repairs.
On Saturday, Minowa repaired 9 nets. How much money did she earn?

Fishing Camp, Ten Mile Lake, Yukon

Here are two strategies to find out.

- Make a table of values.
  Use the patterns in the columns.
  When we add 1 to the number of nets,
  we add $8 to the amount earned.

  The pattern in the number of nets is:
  Start at 0. Add 1 each time.
  The pattern in the amount earned is:
  Start at 25. Add 8 each time.

  We can use these patterns to extend the table.
  Minowa earned $97 for repairing 9 nets.

Number of Fishing Nets	Amount Earned ($)	
0	25	
1	33	8
2	41	8
3	49	8
4	57	8
5	65	8
6	73	8
7	81	8
8	89	8
9	97	8

- Use a variable in an expression.
  Minowa earns $25 even when there are
  no nets to be repaired.
  For each net Minowa repairs, she earns $8.

  For 0 nets, she earns: $8 \times 0 + 25 = 25$
  For 1 net, she earns: $8 \times 1 + 25 = 33$
  For 2 nets, she earns: $8 \times 2 + 25 = 41$
  For 3 nets, she earns: $8 \times 3 + 25 = 49$
  This pattern continues.

  We can use an expression to write the pattern rule.
  We use the letter n to represent any number of nets.
  Then, the amount earned in dollars for repairing n nets is:
  $8 \times n + 25$, or $8n + 25$

  To check that this expression is correct,
  substitute $n = 3$.
  $8n + 25 = 8 \times 3 + 25$
  $= 49$

  This is the same as the amount earned
  for 3 nets in the list above.

  To find the amount earned for repairing 9 nets,
  substitute $n = 9$ into the expression:
  $8n + 25 = 8 \times 9 + 25$
  $= 72 + 25$
  $= 97$

  Minowa earned $97 for repairing 9 nets.

Practice

1. Kilee builds model cars.
She needs 4 plastic wheels for each car she builds.

a) Make a table to show the number of wheels needed for 1, 2, 3, 4, and 5 cars.

b) Write a pattern rule that relates the number of cars to the number of wheels.

c) Write an expression to represent the pattern.

d) Find the number of wheels needed to build 11 cars.
How can you check your answer?

2. For each table of values, write an expression that relates the input to the output.

a)

Input	Output
1	0
2	2
3	4
4	6
5	8

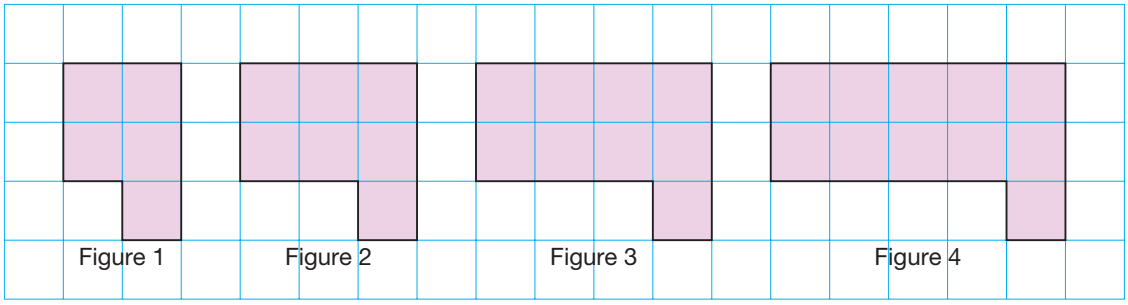
b)

Input	Output
1	5
2	8
3	11
4	14
5	17

c)

Input	Output
1	2
2	6
3	10
4	14
5	18

3. Here is a pattern of squares on grid paper.



a) Make a table to show the numbers of squares in the first 4 figures.

b) Write a pattern rule that relates the figure number to the number of squares.

c) Write an expression to represent the pattern.

d) Find the number of squares in the 7th figure.
Which strategy did you use?
Continue the pattern to check your answer.

4. The Grade 6 class held a dance-a-thon to raise money to buy a new computer for the class. Tyson's friend, Alana, pledged $10, plus $2 for each hour Tyson danced.
   a) Make a table to show the amount Alana pledged for 1, 2, 3, 4, and 5 hours danced.
   b) Write a pattern rule that relates the amount pledged to the number of hours danced. Show your work.
   c) Write an expression to represent the pattern.
   d) Find how much Alana pledged when Tyson danced 9 h. What strategy did you use?
   e) Suppose Alana pledged $34. How many hours did Tyson dance? How did you find out?

5. The pattern in this table continues.
   a) Write a pattern rule that relates the number to the amount.
   b) Write an expression to represent the pattern.
   c) Write a story problem you could solve using the pattern. Solve your problem.

Number	Amount ($)
0	5
1	11
2	17
3	23
4	29

6. Skylar wants to adopt a whale through the BC Wild Killer Whale Adoption Program. The cost of a 1-year adoption is $59. Skylar walks his neighbour's dog to raise the money. He gets $3 for each walk.
   a) Make a table to show the amount left to raise after 1, 2, 3, 4, and 5 walks.
   b) Write a pattern rule that relates the number of walks to the amount left to raise.
   c) Write an expression to represent the pattern.
   d) Find the amount left to raise after 15 walks.
   e) After how many walks will Skylar have raised enough money? How do you know?

Reflect

What is one advantage of using a variable to represent a pattern? How does this help you solve a problem?

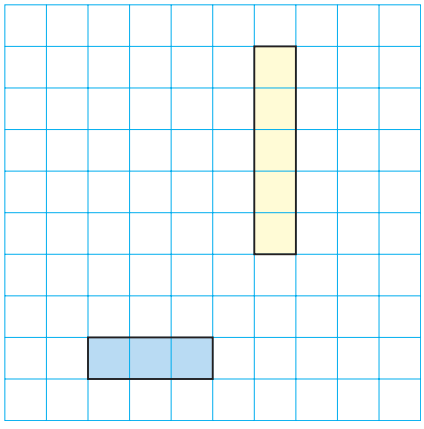
LESSON 5

Plotting Points on a Coordinate Grid

In math, we illustrate ideas whenever we can. To find a way to illustrate Input/Output tables, we need a way to describe the position of a point on a grid.

Explore

Each of you will need two 10 by 10 grids and a ruler.



- Draw a horizontal and a vertical rectangle on your grid.
  Use the grid to the right as an example.
  Place your rectangles where you like.
  Do not show your partner your grid.
- Think of a way to describe the locations of the rectangles to your partner.
- Take turns. Use your method to describe the locations of your rectangles to your partner.
  Your partner uses your description to draw the rectangles on a blank grid. Compare grids. Do they match?
  If not, try to improve your descriptions of the locations.

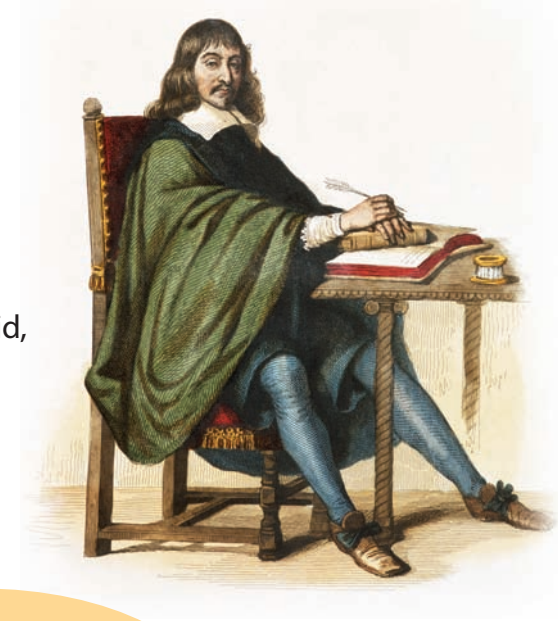
Lesson Focus | Identify and plot points in the Cartesian plane.

Show *and* Share

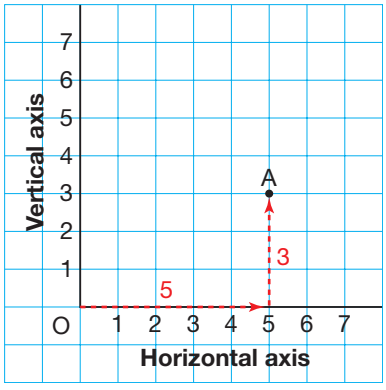
Share your descriptions with another pair of students.
Did you use the same method to describe the locations of the rectangles?
If your answer is no, do both methods work?

Connect

René Descartes was a French mathematician who lived from 1596 to 1650.
He developed the **coordinate grid** system shown below.
In his honour, it is called the **Cartesian plane**.



➤ Two perpendicular number lines intersect at 0.
The point of intersection, O, is called the **origin**.
To describe the position of a point on a coordinate grid, we use two numbers.
The numbers locate a point in relation to the origin, O.



The first number tells how far you move right.
The second number tells how far you move up.

From O, to reach point A, we move 5 units right and 3 units up.
We write these numbers in brackets: (5, 3)
These numbers are called **coordinates**.
Because the coordinates are always written in the same order, the numbers are also called an **ordered pair**.

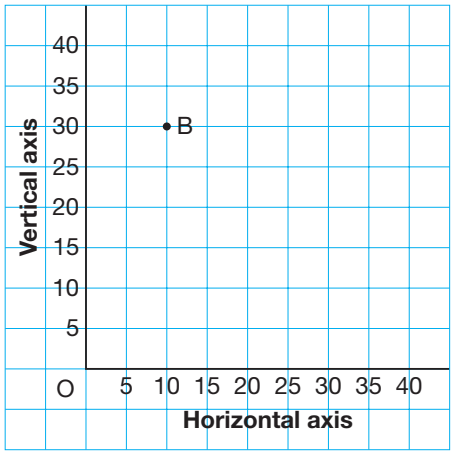
We move right along the **horizontal axis**.
We use the **vertical axis** to count the units up.

We say: A has coordinates (5, 3).
We write: A(5, 3)

The point O has coordinates (0, 0) because you do not move anywhere to plot a point at O.

➤ When the numbers in an ordered pair are large, we use a scale on the coordinate grid.
On this coordinate grid, 1 square represents 5 units.

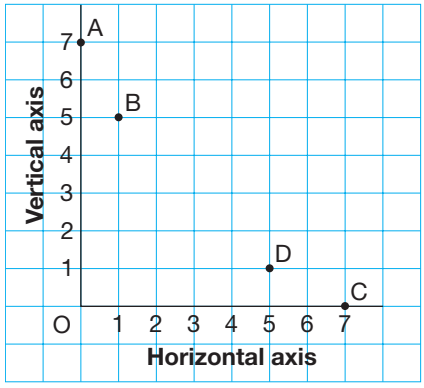
To plot point B(10, 30):
Start at O.
Move 2 squares right.
Move 6 squares up.



Practice

1. Match each ordered pair with a letter on the coordinate grid.

a) (1, 5)
b) (5, 1)
c) (0, 7)
d) (7, 0)



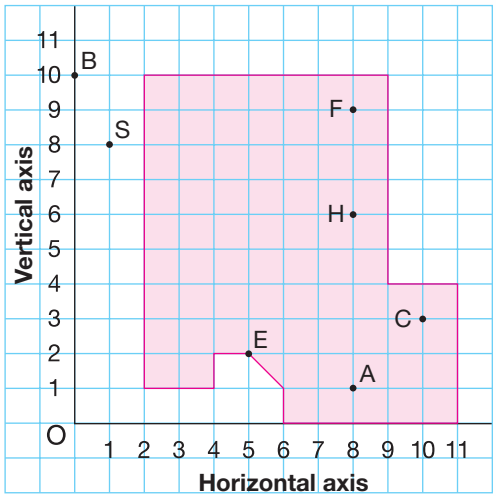
2. Draw and label a coordinate grid.
Plot each ordered pair.
Explain how you moved to do this.

a) V(5, 9) **b)** W(0, 9) **c)** X(5, 7) **d)** Y(8, 0)

3. Draw and label a coordinate grid.
Plot each point on the grid.

a) P(2, 7) **b)** Q(6, 5) **c)** R(1, 4) **d)** S(0, 3) **e)** O(0, 0)

4. Mr. Kelp's class went to the Vancouver Aquarium.
Angel drew this map of the aquarium site.



Write the ordered pair for each place.

a) Amazon Jungle Area: A
b) Beluga Whales: B
c) Carmen the Reptile: C
d) Entrance: E
e) Frogs: F
f) Sea Otters: S
g) Sharks: H

5. Use the map in question 4.

a) To get to the Pacific Canada Pavilion at point P:
You move 1 square left and 3 squares up from the entrance, E.
What are the coordinates of P?

b) To get to the Clam Shell Gift Shop at point G:
You move 5 squares left and 4 squares down from the sharks, H.
What are the coordinates of G?

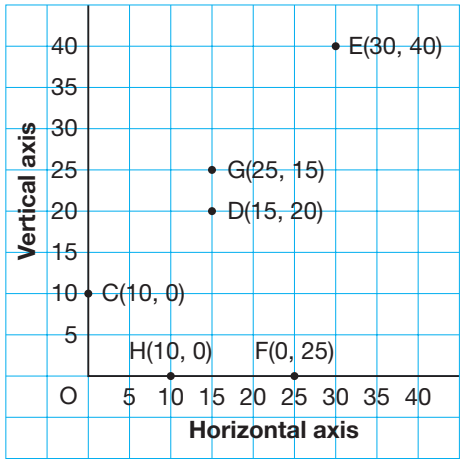
6. Draw and label a coordinate grid.
Plot each point on the grid.
How did you decide which scale to use on the axes?

a) A(10, 40) **b)** B(10, 0) **c)** C(20, 20) **d)** D(0, 30) **e)** E(50, 60)

7. Draw and label a coordinate grid.
Plot each point on the grid.
How did you decide which scale to use on the axes?

a) J(14, 20) **b)** K(6, 12) **c)** L(0, 18) **d)** M(8, 4) **e)** N(16, 0)

8. A student plotted 6 points on a coordinate grid, then labelled each point with its coordinates. The student has made some mistakes. For each point that has been labelled incorrectly:
   a) Explain the mistake.
   b) Write the coordinates that correctly describe the location of the point.



9. Draw and label a coordinate grid.
   Use a scale of 1 square represents 5 units.
   Plot 5 points on the grid.
   Use an ordered pair to describe the location of each point.

10. **a)** The first number in the ordered pair for Point A is 0. What does this tell you about Point A?
    b) The second number in the ordered pair for Point B is 0. What does this tell you about Point B?

Math Link

Agriculture

To maximize crop yield, farmers test the soil in their fields for nutrients. The results help farmers to decide on the amount and type of fertilizer to use. Grid soil sampling is one method of collecting samples. The field is divided into a grid. A soil sample is taken from the centre of each grid cell.

Reflect

How is plotting a point on a coordinate grid similar to plotting a point on a number line? How is it different?

At Home

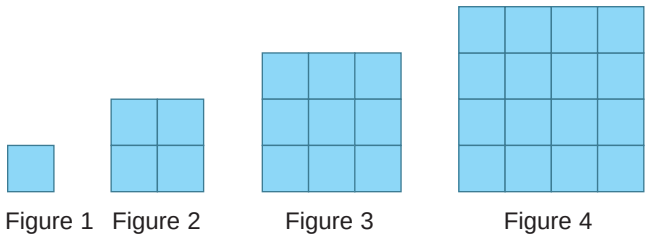
Look at a map of your neighbourhood. Suppose a delivery truck is trying to find your home. How would you use the map to describe the location of your home to the driver?

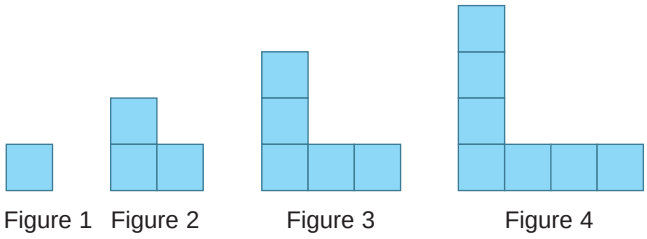
LESSON 6

Drawing the Graph of a Pattern

How are these patterns alike?
How are they different?

Describe Figure 5 for each pattern.

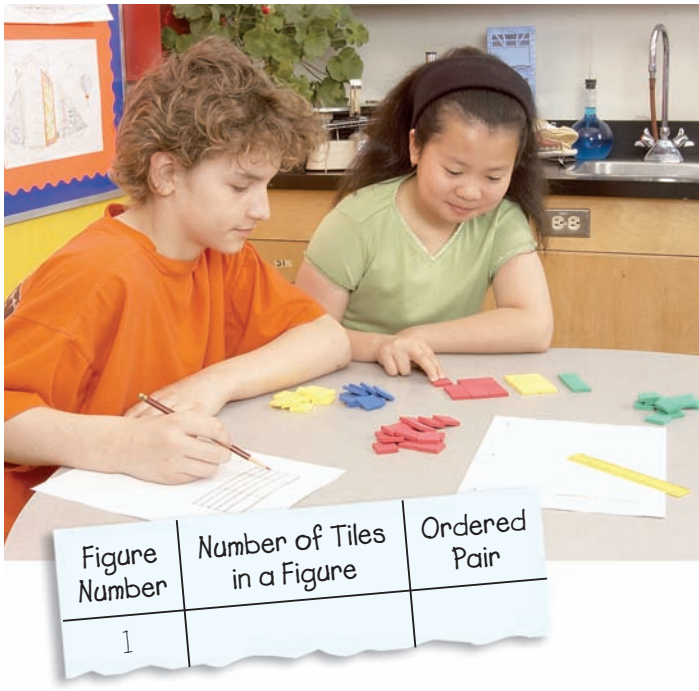




Explore

You will need Colour Tiles or congruent squares, and grid paper.

- Use Colour Tiles.
  Build the first 4 figures of a growing pattern.
  Record your pattern on grid paper.
- Make a table.
  Record each figure number and its number of tiles.
  Write these numbers as an ordered pair.
- Plot each ordered pair on a coordinate grid.
  Describe the graph formed by the points.



Show *and* Share

Share your work with another pair of students.
Compare your graphs.
If they are different, try to find out why.

Connect

➤ Here are some different ways to represent a pattern.

- Model the pattern with tiles or on grid paper.

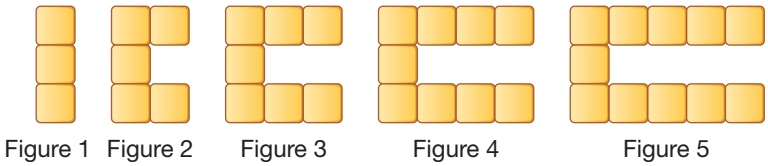


Figure 1 Figure 2 Figure 3 Figure 4 Figure 5

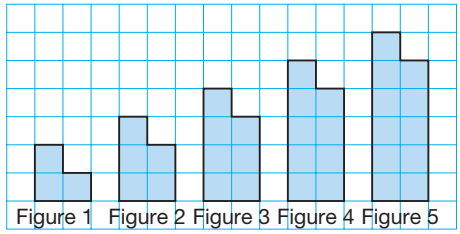


Figure 1 Figure 2 Figure 3 Figure 4 Figure 5

- Make a table. Include a column for ordered pairs.

Figure Number	Number of Tiles	Ordered Pair
1	3	(1, 3)
2	5	(2, 5)
3	7	(3, 7)
4	9	(4, 9)
5	11	(5, 11)
6	13	(6, 13)
7	15	(7, 15)

The figure number is the first coordinate. The number of tiles in a figure is the second coordinate.

We have extended the table to find the number of tiles in the 7th figure.

- Draw a graph.
  Draw and label a coordinate grid.

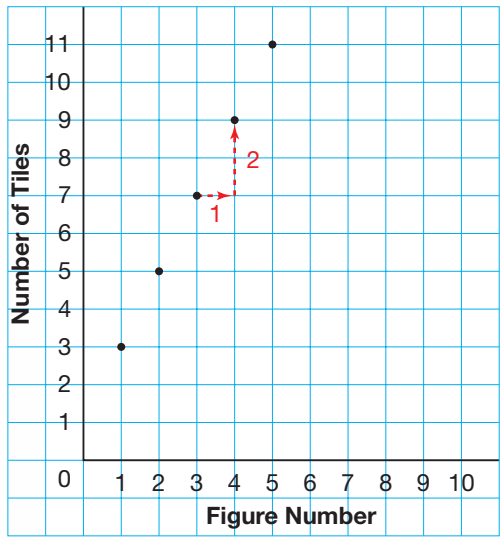
We label the axes with the column headings.

Plot the ordered pairs.
Mark points at (1, 3), (2, 5), (3, 7), (4, 9), and (5, 11).

From the graph, we see that each time the figure number increases by 1, the number of tiles increases by 2.

From (3, 7), move 1 to the right and 2 up to reach (4, 9).

Number of Tiles in a Pattern

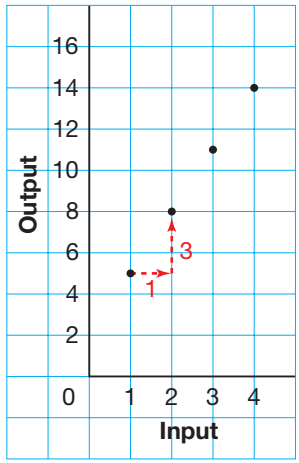


To get from one point to the next, move 1 to the right and 2 up.

➤ We can graph the relationship shown in an Input/Output table.

Input	Output
1	5
2	8
3	11
4	14

As the input increases by 1, the output increases by 3.

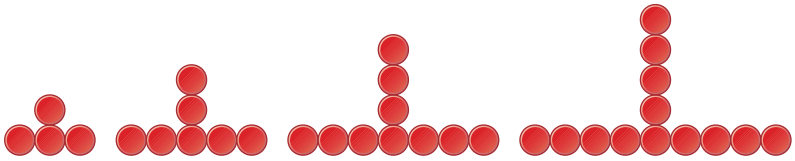


Practice

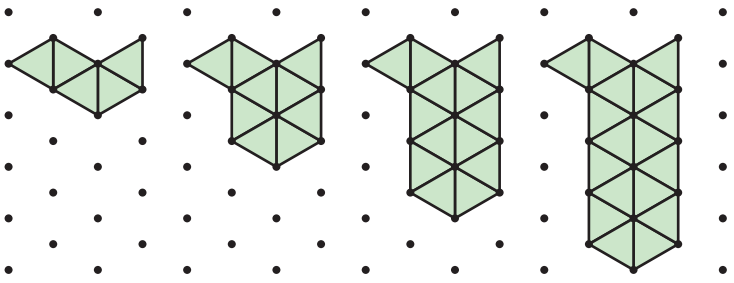
1. Record each pattern in a table. Then draw a graph to represent the pattern. Explain how the graph represents the pattern.

a)

b)



c)



2. Use grid paper. Graph each table.
Describe the relationship shown on the graph.

a)

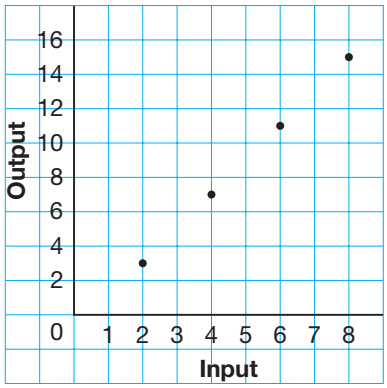
Input	Output
1	3
2	6
3	9
4	12

b)

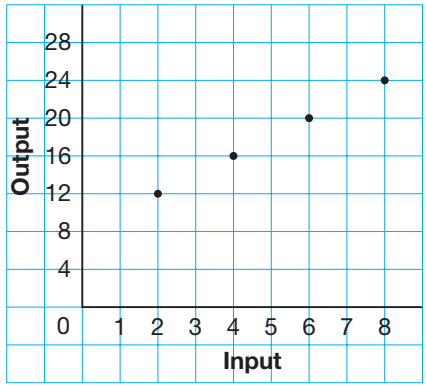
Input	Output
1	5
2	6
3	7
4	8

3. For each graph, make an Input/Output table.

a)



b)



4. Use grid paper.

a) Graph the data in the table.

b) Describe the relationship shown on the graph.

c) Write an expression to represent the pattern.

d) Find the number of shapes in the 8th figure.
What strategy did you use?
Could you use the same strategy to find the number of shapes in the 18th figure?
Explain.

Figure Number	Number of Shapes
1	1
2	6
3	11
4	16
5	21

5. Use grid paper.

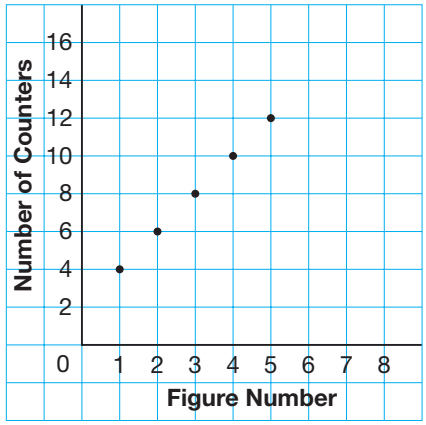
a) Make a table.
Record the figure number and the number of counters in a figure.

b) How does the graph represent the pattern?

c) Find the number of counters in the 7th figure.
Describe the strategy you used.

d) How many counters are in the 23rd figure?
Describe the strategy you used to find out.

Number of Counters in a Pattern



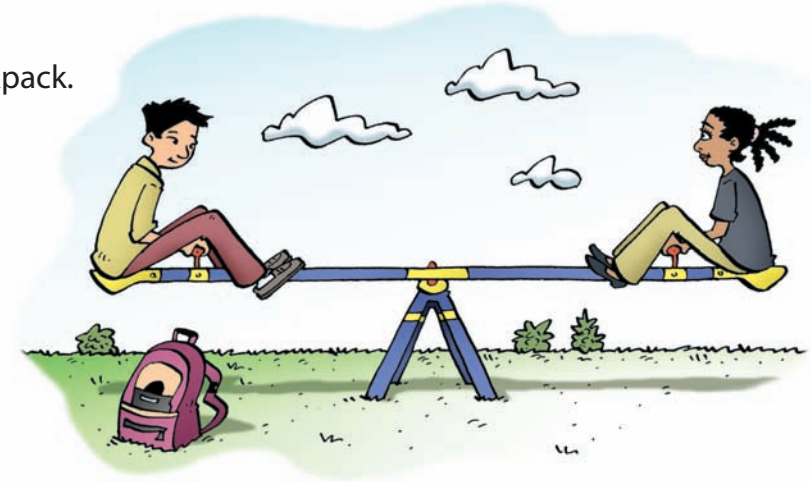
Reflect

Describe some of the different ways you can represent a pattern.
Which way do you prefer? Why?

LESSON 7

Understanding Equality

Suppose the boy puts on his backpack.
What will happen?



Explore

You will need balance scales, counters, and drawings of balance scales.

- Choose 2 expressions from the box at the right.
  On a drawing of balance scales, write one expression in each pan.
- Suppose you were using real balance scales and counters for the numbers.
  Would the scales tilt to the left, to the right, or would they balance?
  How do you know?
  Use balance scales and counters to check.
- Repeat the steps above with different pairs of expressions. Find as many pairs of expressions as you can that balance.

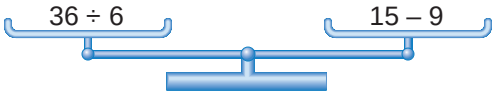
Expressions	
$4 + 5$	$8 + 3$
3×5	2×4
$17 - 10$	4×2
$18 \div 6$	$24 \div 4$
$15 - 8$	$30 \div 5$
$21 - 10$	$5 + 4$
$27 \div 9$	5×3

Show *and* Share

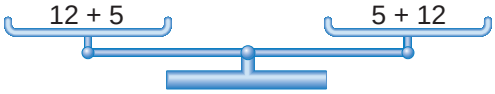
Share your work with another pair of classmates.
What strategies did you use to decide whether the scales balance or tilt?
What did you notice about the expressions $4 + 5$ and $5 + 4$, and 2×4 and 4×2?
What does it mean when the scales balance?

Connect

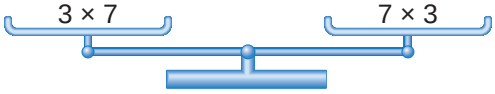
Each of the scales below are balanced.
For each balance scales, the expression in one pan is equal to the expression in the other pan.
We use the equals sign to show that the two expressions are equal.



$36 \div 6 = 6$ and
$15 - 9 = 6$
So, $36 \div 6 = 15 - 9$



$12 + 5 = 17$ and
$5 + 12 = 17$
So, $12 + 5 = 5 + 12$



$3 \times 7 = 21$ and
$7 \times 3 = 21$
So, $3 \times 7 = 7 \times 3$

➤ When we add 2 numbers, their order does not affect the sum.
The scales always balance.
This is called the **commutative property of addition**.
For example,

$3 + 2 = 2 + 3$
$114 + 35 = 35 + 114$

We can use variables to show this property for any pair of numbers we add:

$a + b = b + a$

➤ Multiplication is also *commutative*.
When we multiply two numbers, their order does not affect the product.
For example,

$3 \times 2 = 2 \times 3$
$55 \times 8 = 8 \times 55$

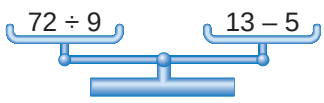
We can use variables to show this property for any pair of numbers we multiply:

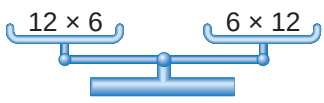
$a \times b = b \times a$

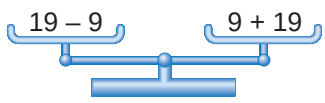
This illustrates the **commutative property of multiplication**.

Practice

1. Suppose you were using real balance scales.
Which scales below would balance?
How did you find out?

a) 

b) 

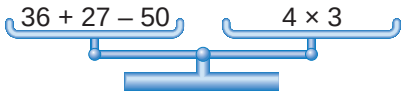
c) 

2. **a)** Write an expression with 2 numbers and one operation.
b) Write 5 different expressions that equal your expression in part a.
What strategy did you use to find the expressions?
c) Suppose you used real balance scales.
You put counters to represent 3 of the expressions in the left pan and 3 in the right pan. What would happen? How do you know?

3. Rewrite each expression using a commutative property.

a) $5 + 8$ **b)** 6×9 **c)** 11×7
d) $12 + 21$ **e)** $134 + 72$ **f)** 36×9

4. **a)** Are these scales balanced?



b) If your answer is yes, why do you think so?
If your answer is no, what could you do to balance the scales?
Why would this work?

5. **a)** Addition and subtraction are inverse operations.
Addition is commutative. Is subtraction commutative?
Use an example to show your answer.
b) Multiplication and division are inverse operations.
Multiplication is commutative. Is division commutative?
Use an example to show your answer.

Reflect

Are subtractions and division commutative operations?
Explain why or why not.

LESSON 8

Keeping Equations Balanced

Each of these tug-of-war teams has the same total mass.
Suppose a girl with mass 48 kg joins Team A.
What could be done to keep the match fair?

Explore

You will need counters.
Each group member chooses a different expression.

Expressions	
3×6	$17 - 5$
$3 + 5$	$24 \div 4$

- Write a different expression that is equal to the expression you chose.
  Use the expressions to write an equation.
- Model the equation with counters.
  How do the counters show the expressions are balanced?
- Find 4 different ways to adjust the original equation so that it remains balanced.
  Use counters to model what you did each time.
  Use symbols to record your work.

Show *and* Share

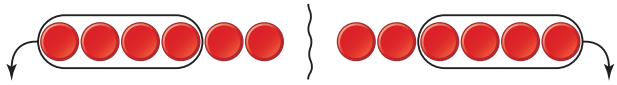
Share your work with another group of students.
What strategies did you use to keep the equation balanced?
Were you able to use each of the 4 operations?
If not, work together to try the operations that you did not use.

Lesson Focus | Model and explain the meaning of the preservation of equality.

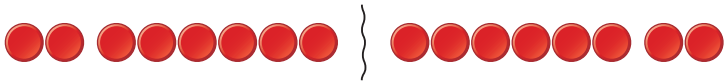
Connect

➤ Max started with this equation each time:
$2 + 4 = 3 \times 2$
He modelled it using counters.
Each side has 6 counters.

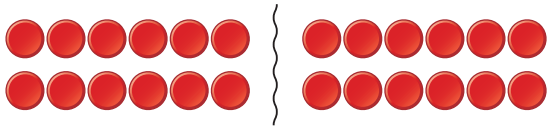
First, Max subtracted 4 from each side.
$6 - 4 = 6 - 4$
Each side now has 2 counters.



Second, Max added 2 to each side.
$6 + 2 = 6 + 2$
Each side now has 8 counters.



Third, Max multiplied each side by 2.
$6 \times 2 = 6 \times 2$
Each side now has 12 counters.



Fourth, Max divided each side into 2 equal groups.
$6 \div 2 = 6 \div 2$
Each group has 3 counters.

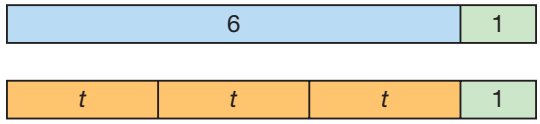
Whatever Max did to one side of the equation, he did to the other side, too.
Each time, the numbers of counters on both sides remained equal.
So, the equation remained balanced.

When each side of the equation is changed in the same way,
the values remain equal.
This is called the **preservation of equality**.
The same is true if one side of the equation is an expression containing a variable.

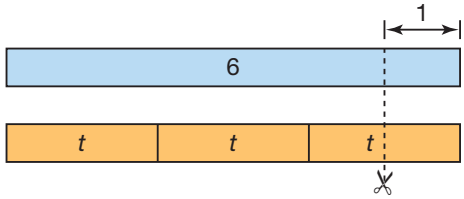
➤ Suppose we know $6 = 3t$.
We can model this equation with paper strips.

To preserve the equality, we can:

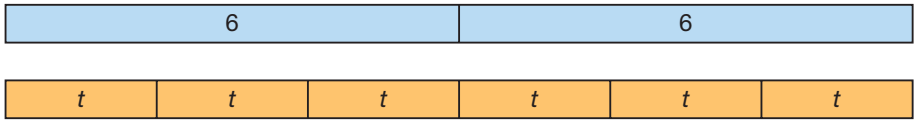
- Add the same number to each side.
  So, $6 + 1 = 3t + 1$



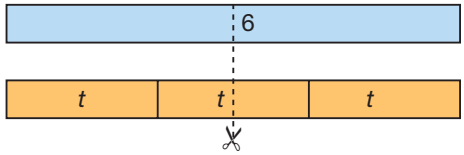
- Subtract the same number from each side.
  So, $6 - 1 = 3t - 1$



- Multiply each side by the same number.
  So, $2 \times 6 = 2 \times 3t$



- Divide each side by the same number.
  So, $6 \div 2 = 3t \div 2$



When we do the same to each side of an equation, we produce an **equivalent form of the equation**.

So, $6 + 1 = 3t + 1$
$6 - 1 = 3t - 1$
$2 \times 6 = 2 \times 3t$
$6 \div 2 = 3t \div 2$
} are all equivalent forms of the equation $6 = 3t$.

Practice

1. For each equation below:

- Model the equation with counters.
- Use counters to model the preservation of equality for addition.
- Draw a diagram to record your work.
- Use symbols to record your work.

a) $9 + 6 = 15$ **b)** $14 - 8 = 6$
c) $2 \times 5 = 10$ **d)** $15 \div 3 = 9 - 4$

2. For each equation below:

- Model the equation with counters.
- Use counters to model the preservation of equality for subtraction.
- Draw a diagram to record your work.
- Use symbols to record your work.

a) $7 + 8 = 15$ **b)** $12 - 7 = 5$

c) $3 \times 4 = 12$ **d)** $10 \div 5 = 9 - 7$

3. For each equation below:

- Model the equation with counters.
- Use counters to model the preservation of equality for multiplication.
- Draw a diagram to record your work.
- Use symbols to record your work.

a) $2 + 3 = 5$ **b)** $9 - 6 = 3$

c) $2 \times 4 = 8$ **d)** $12 \div 4 = 2 + 1$

4. For each equation below:

- Model the equation with counters.
- Use counters to model the preservation of equality for division.
- Draw a diagram to record your work.
- Use symbols to record your work.

a) $5 + 1 = 6$ **b)** $8 - 4 = 4$

c) $5 \times 2 = 10$ **d)** $16 \div 2 = 2 \times 4$

5. For each equation below:

- Apply the preservation of equality.
  Write an equivalent form of the equation.
- Use paper strips to check that equality has been preserved.

Try to use a different operation for each part.

a) $3b = 12$ **b)** $2t = 8$

c) $16 = 4s$ **d)** $15 = 5s$

How do you know that equality has been preserved each time?

Reflect

Talk to a partner. Tell your partner what you think the preservation of equality means. Describe how you could model the preservation of equality for each of the 4 operations.

Unit 1 Show What You Know

LESSON

1

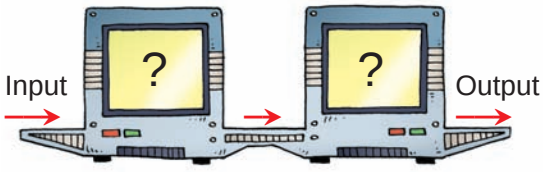
1. The pattern rule that relates the input to the output is:
Divide the input by 5, then subtract 1.

Input	Output
5	0
10	2
15	3
30	7
45	8
50	11

a) Check the data in the table.
Identify any output numbers that are incorrect.
How do you know they are incorrect?
b) Write the pattern rule for the input.
c) Write the pattern rule for the corrected output.
d) The pattern continues.
Write the next 4 input and output numbers.

2

2. The table shows the input and output for this machine.



Input	Output
1	0
2	2
3	4
4	6
5	8
6	10
7	12

a) Identify the numbers and operations in the machine.
b) Write a pattern rule that relates the input to the output.
c) Choose 4 different input numbers.
Find the output for each input.
d) Predict the output when the input is 11. Check your prediction.

4

3. In a dogsled race, teams of 6 dogs race to the finish.

a) Make a table to show the numbers of dogs in a race when 2, 3, 4, 5, and 6 teams are entered.
b) Write a pattern rule that relates the number of dogs to the number of teams entered.
c) Write an expression to represent this pattern.
d) Use the expression to find the number of dogs when 13 teams are entered.
How can you check your answer?

5

4. Draw and label a coordinate grid.
Plot each point on the grid.
How did you decide which scale to use on the axes?
a) A(10, 5) **b)** B(0, 20) **c)** C(20, 30) **d)** D(0, 0) **e)** E(30, 0)

LESSON

6

5. Use dot paper.

a) Draw a pattern to model the data in the table. Extend the pattern to Figure 6.

b) Graph the data in the table.

c) Describe the relationship shown on the graph.

d) Write an expression to represent the pattern.

e) Find the number of shapes in the 21st figure. Which strategy did you use? Why?

Figure Number	Number of Shapes
1	4
2	8
3	12
4	16

7

6. Rewrite each expression using a commutative property.

a) 24×3 **b)** $121 + 27$

c) $46 + 15$ **d)** 9×12

e) 11×8 **f)** $37 + 93$

8

7. For each equation below:

- Model the equation with counters.
- Use counters to model the preservation of equality. Use a different operation for each equation.
- Draw diagrams to record your work.
- Use symbols to record your work.

a) $11 - 3 = 8$

b) $3 \times 1 = 5 - 2$

c) $3 + 4 = 7$

d) $12 \div 6 = 9 - 7$

8. For each equation below:

- Apply the preservation of equality. Write an equivalent form of the equation.
- Use paper strips to check that equality has been preserved.

Try to use a different operation for each part.

a) $4b = 8$

b) $t = 3$

c) $12 = 6s$

d) $4 = 2s$

How do you know that equality has been preserved each time?

UNIT 1 Learning Goals

- ☑ describe patterns and relationships using graphs and tables
- ☑ use equations to represent number relationships
- ☑ use relationships within tables of values to solve problems
- ☑ identify and plot points in a Cartesian plane
- ☑ demonstrate the preservation of equality

Unit Problem

Crack the Code!

Jen and Rodrigo are planning a surprise skating party for their friend Lacy.
They use a secret code to send messages to each other.

To create their code, Jen and Rodrigo wrote the position number of each letter in the alphabet.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

They applied a secret pattern rule to each number.
Then, each letter is represented by a code number.

Jen's copy of their code went through the washing machine.
Here is what was left of the code.

A is represented by 1.
B is represented by 8.

Step 1 Copy and complete the table for the first 8 letters of the alphabet.

- Write the pattern rule for the position number.
- Write the pattern rule for the code number.
- Write the pattern rule that relates the position number to the code number. Write the rule in words and using symbols.
- Which code number represents the letter "Y" in a message? Can you find this code number without completing the table for the entire alphabet? Explain.

Step 2 Here is a coded message that Jen received from Rodrigo. What does it say?

155 50 1 134	134 57 85 29
155 57 78 78	134 50 29
106 1 120 134 169	127 134 1 120 134?

Step 3 Jen replies to Rodrigo with a mystery picture.
To see Jen's reply, draw and label a 10 by 10 coordinate grid.
Plot these points on the grid.
Join the points in order. Then join the last point to the first point.
(3, 7), (6, 7), (6, 2), (3, 2), (3, 3), (5, 3), (5, 4), (4, 4), (4, 5), (5, 5), (5, 6), (3, 6)

Step 4 Work with a partner. Make up your own code for the letters of the alphabet.

- Make a table to show the code for the first 5 letters of the alphabet.
- Describe the pattern rules for the position number and code number.
- Describe the pattern rule that relates the position number to the code number.
- Write an expression to represent the pattern.
- Represent the pattern on a graph. Describe how the graph represents the pattern.
- Write messages to each other using your code.

Check List

Your work should show

- ☑ completed tables
- ☑ pattern rules represented in words and in symbols
- ☑ the decoded message
- ☑ a graph that represents your code
- ☑ clear descriptions using math language

Reflect on Your Learning

What did you find easy about working with patterns?
What was difficult for you?
Give examples to show your answers.