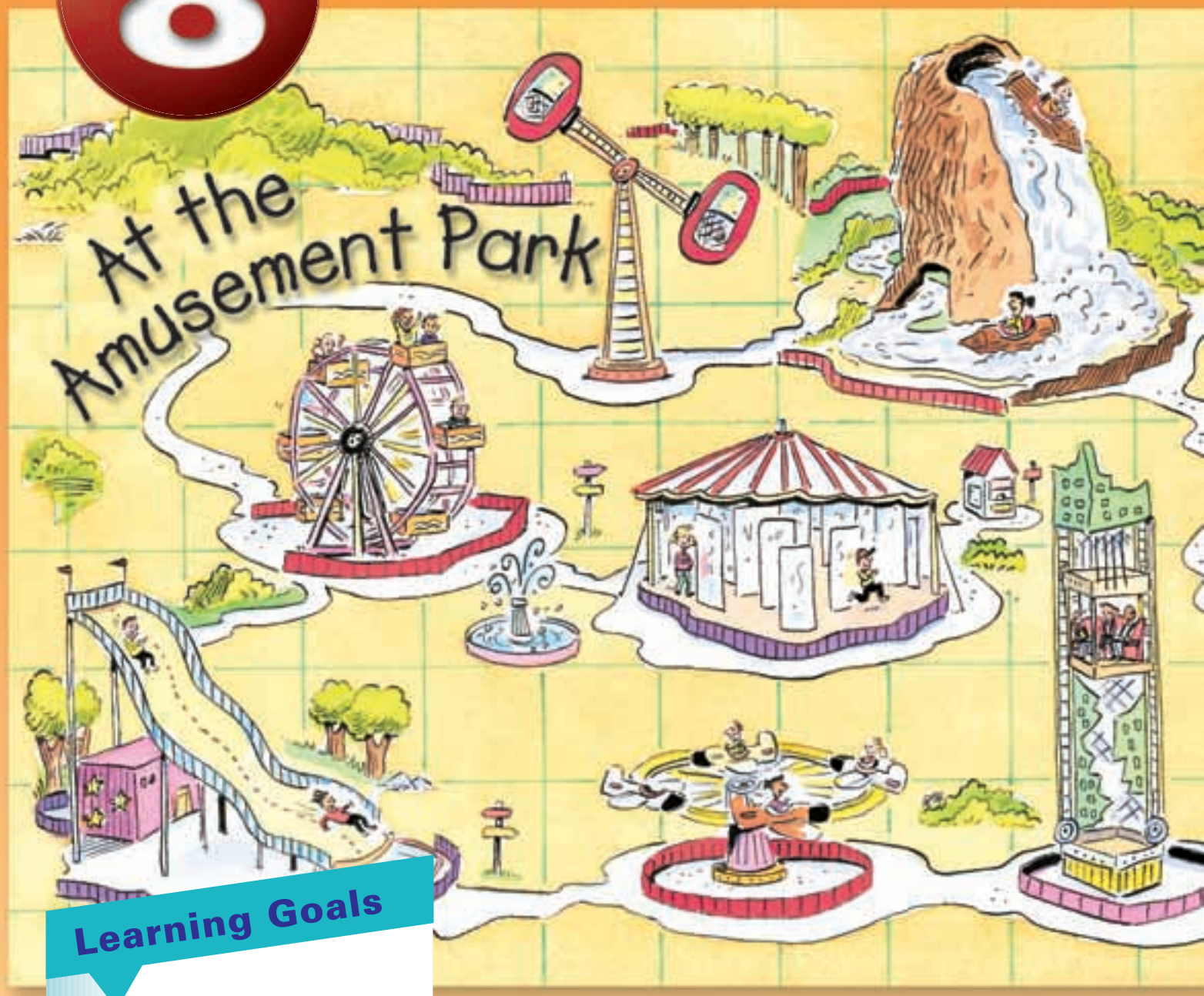
UNIT 8

Transformations

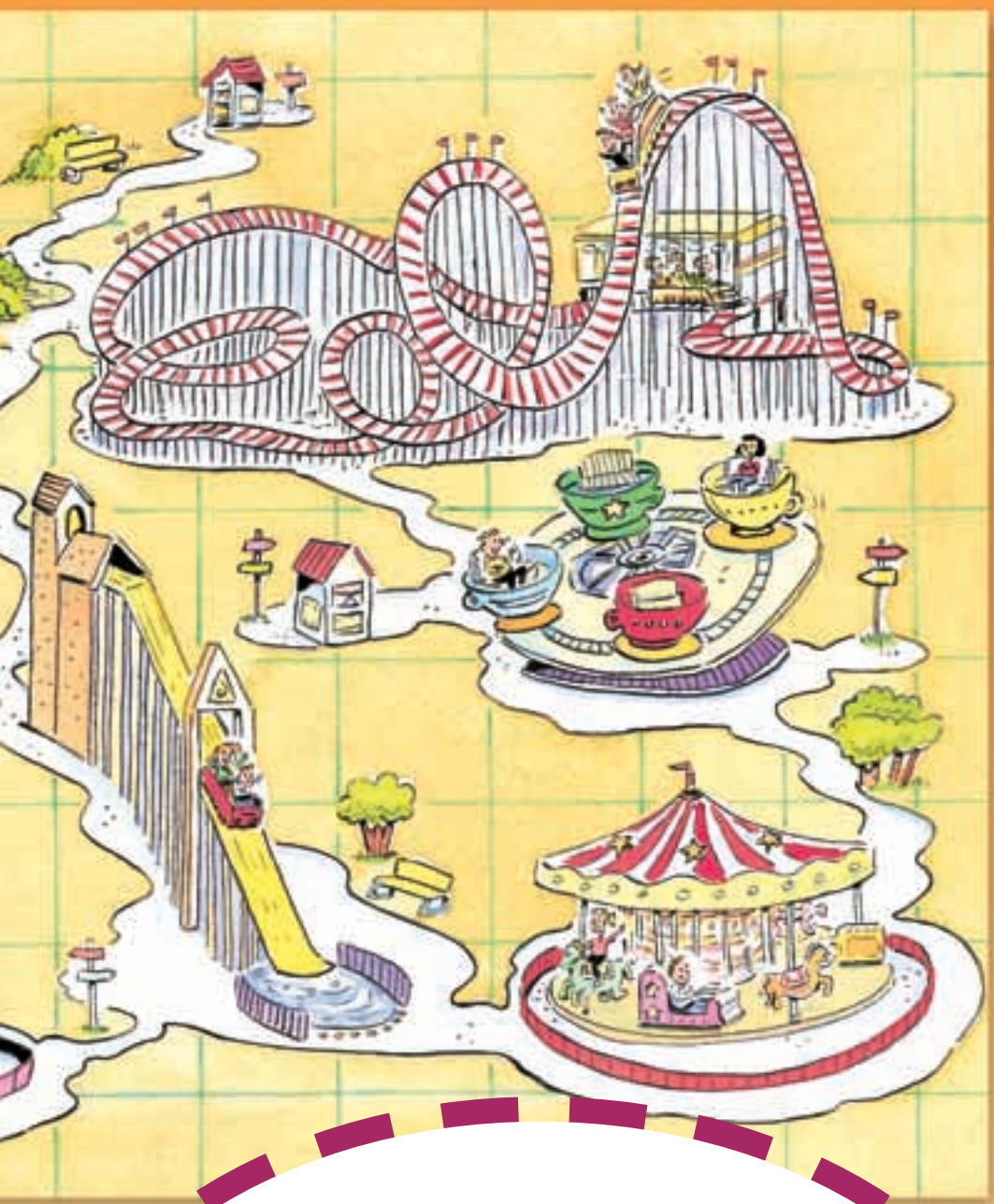


Learning Goals

- translate, reflect, and rotate a shape
- draw and describe images after transformations
- identify a transformation

Key Words

- translation
- translation image
- translation arrow
- reflection
- reflection image
- line of reflection
- rotation
- rotation image
- quarter turn
- half turn
- three-quarter turn
- clockwise
- counterclockwise
- point of rotation
- transformations



Look at the map of the amusement park.

- What rides do you see?
- How do people move on each ride?
- What is your favourite ride at an amusement park? How do you move on that ride?

LESSON 1

Translations

A firefighter slides down a pole.

A flag slides up a pole.

A child slides down a playground slide.

Which other ways do people or objects slide?

Explore

You will need Pattern Blocks, dot paper, and a ruler.

- Choose a Pattern Block.
  Place it on the dot paper.
  Trace the block.
  Slide the block in a straight line, in any direction.
  Do not turn the block.
  Use a ruler if it helps.
  Trace the block in its new position.
  How do the two positions of the block compare?

- Take turns to move a block and describe how its two positions compare.

Show *and* Share

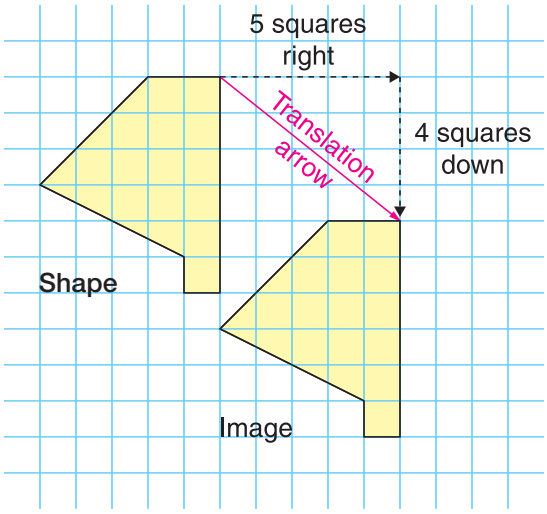
Compare drawings with another pair of classmates.
How do the original shape and the shape in its new position compare?

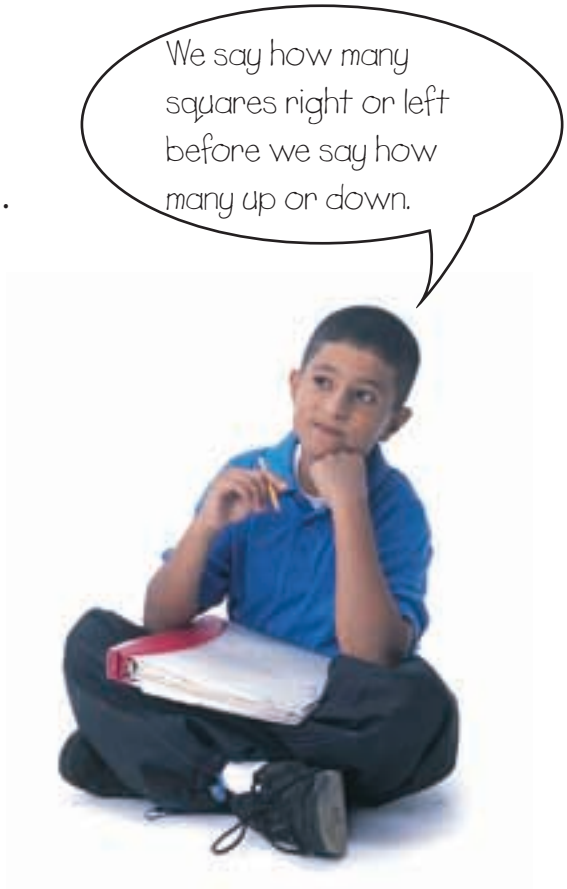
Connect

When a shape moves along a straight line, without turning,
it is *translated* from one position to another.
The movement is a **translation** or a *slide*.

When we draw the shape in its new position,
we draw a **translation image** of the shape.

The translation is described by the numbers
of squares moved right or left and up or down.
The translation below is:
5 squares right and 4 squares down.





If we cannot translate the shape, we trace the shape,
then translate the tracing.

The **translation arrow** shows how the shape moved.
The arrow joins matching points on the shape and its image.

A shape and its translation image have the same orientation;
that is, they face the same way.

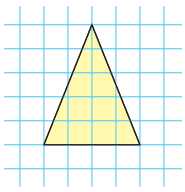
Practice

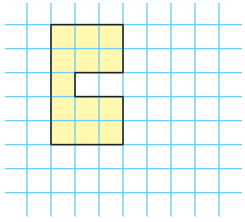
1. Copy each shape on grid paper. Use tracing paper.
Translate the shape using the given translation.
Draw the image and a translation arrow.
Describe the position and orientation of the image.

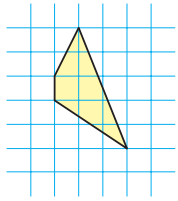
a) 7 squares left and 3 squares up

b) 5 squares right and 4 squares down

c) 3 squares left and 6 squares down

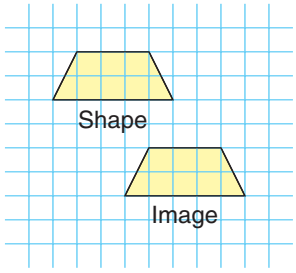






2. Does each picture show a translation?
How do you know?
If a picture does show a translation, describe it.

a)



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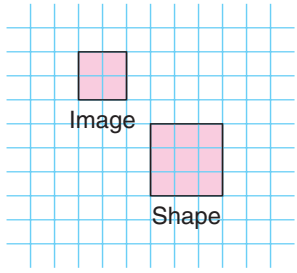

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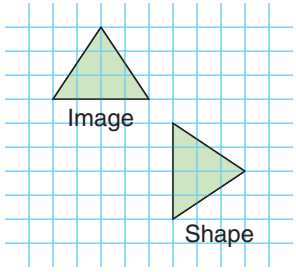
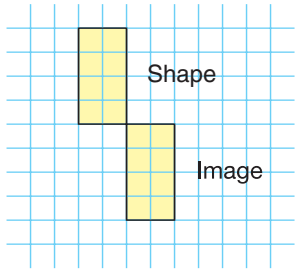
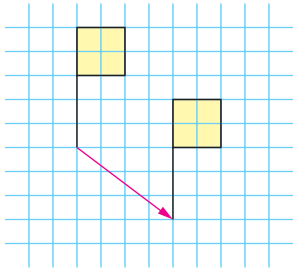


d)



3. Write the translation that moved each shape to its image.

a)



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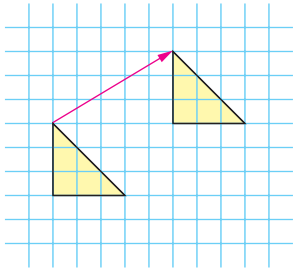

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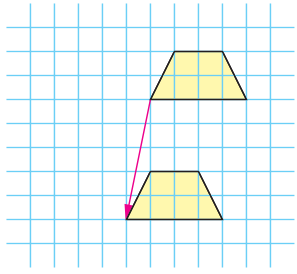
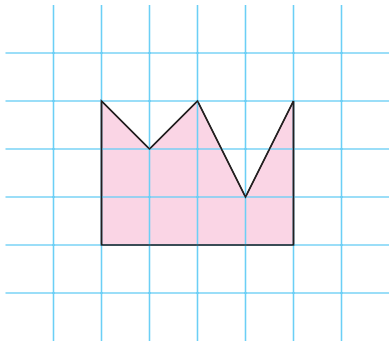


4. **a)** Draw this shape on grid paper.
Predict where the image will be after this translation:
3 squares left and 5 squares up
Draw the image to check your prediction.

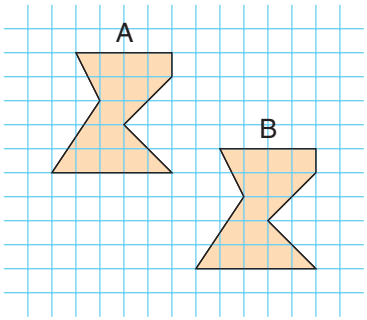


b) Draw the shape again.
Predict where the image will be after this translation:
5 squares left and 3 squares up
Draw the image to check your prediction.

5. Draw a shape on dot paper.
Translate the shape in any direction.
Draw its image.
Record the translation.
Describe the position and orientation of the image.

6. Draw two identical shapes in two different places on grid paper.
Make sure the shapes have the same orientation.
Label one shape "Image" and the other "Shape".
Which translation will move the shape to its image?

7. Copy these shapes on grid paper.



a) Describe which translation moves Shape A to Shape B.
b) Describe which translation moves Shape B to Shape A.

Reflect

Use grid paper.
Draw a shape and its translation image.
Explain how you know your picture shows a translation.

LESSON 2

Strategies Toolkit

Explore

The hare and the tortoise had a race.
The race was 5 times around the running track.
The hare ran 4 times around in 1 h,
then stopped for a rest.
The tortoise did not stop.
She took 1 h to go once around the track.
The hare woke up after 4 h.
Who won the race?

Show *and* Share

Explain how you solved the problem.

Connect

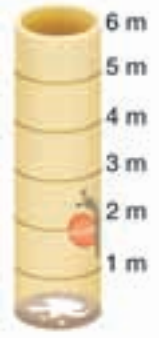
A snail is at the bottom of a well.
It climbs 2 m every day, but it slides
back 1 m at night.
The well is 6 m deep.
How many days does it take the snail
to get out of the well?

What do you know?

- Each day, the snail climbs 2 m up the well.
- Each night, the snail slides back 1 m.
- The snail has to climb 6 m to get out.

Think of a strategy to help you solve the problem.

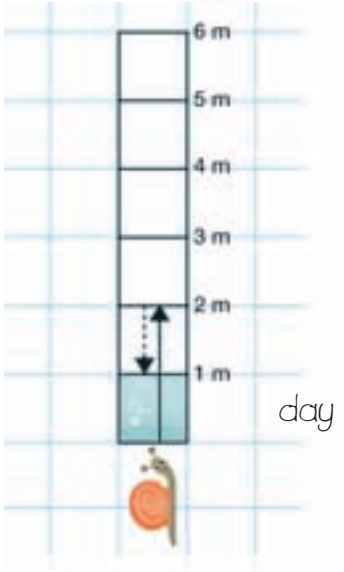
- You can **draw a picture.**
- Show where the snail is each day.



Strategies

- **Make a table.**
- **Use a model.**
- **Draw a picture.**
- **Solve a simpler problem.**
- **Work backward.**
- **Guess and test.**
- **Make an organized list.**
- **Use a pattern.**

Use grid paper to record how far the snail moves.
Use a different colour for each day.
Count the days when the snail reaches the top of the well.
When does the snail get out of the well?



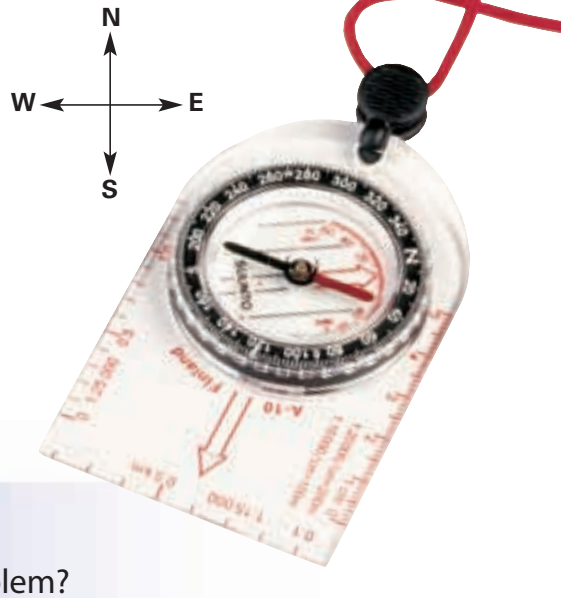
Write a similar problem.
Have a classmate solve your problem.

Practice

Choose one of the **Strategies**

1. Shannon is shorter than Bruce.
Olivia is shorter than Alex but taller than Bruce.
Who is the tallest? Shortest?

2. Hannah and Liam are using a compass.
They move 30 m north, then 30 m west, and then 30 m south.
Which direction do Hannah and Liam go to get back to where they started?
How far must they go?



Reflect

How does drawing a picture help you to solve a problem?

LESSON 3

Reflections

A **reflection** can be used to make an interesting picture.
Is this person floating above the ground?
Where else do you see reflections?

Explore

You will need Pattern Blocks, dot paper, a ruler, and a Mira.

- Draw a line through the centre of the dot paper.
  Place a Mira on this line.
- Place a block on one side of the line.
  Your partner places her block on the image she sees in the Mira.
- Take turns to place one block, then another block on its image.
  Each time, describe the position and orientation of the image.

➤ Take turns to draw around a block and its image.
Draw around blocks that touch the Mira line.
Draw around blocks that cross the Mira line.
In each case, how does the shape compare with its image?

Show *and* Share

Compare your pictures with those of another pair of classmates.
How is each shape and its image placed with respect to the Mira line?

Connect

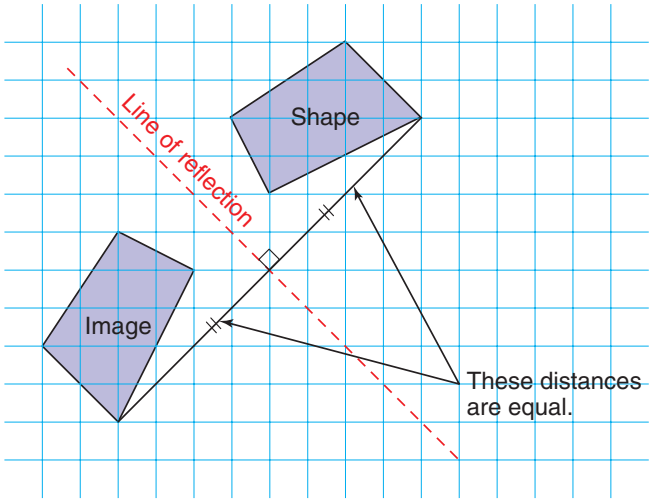
When a shape is *reflected* in a mirror, we see a **reflection image**.

The line segment that joins a point to its image is perpendicular to the **line of reflection**.

A point and its image are the same distance from the line of reflection.

A shape and its reflection image have opposite orientations; that is, they face opposite ways.

A reflection is sometimes called a *flip*.
When a shape is reflected, it is flipped over.



Math Link

Your World

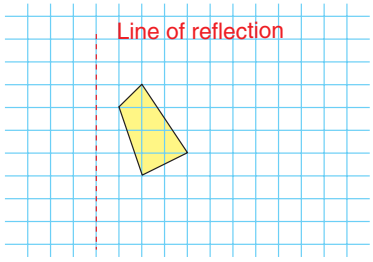
Many patterns and designs show a shape and its reflection images.
Identify a shape and its reflection images in this design.
Where are the lines of reflection?

Practice

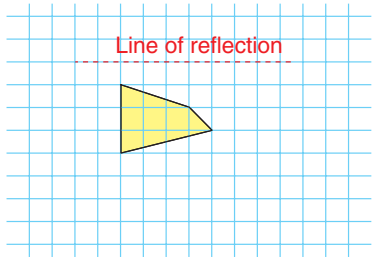
Use a Mira when it helps.

1. Copy each shape and line of reflection on grid paper.
Draw each reflection image.
Describe the position and orientation of the image.

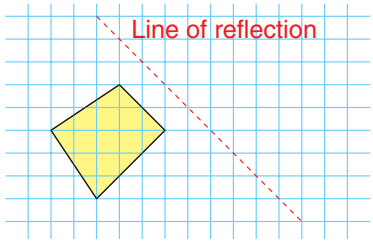
a)



b)



c)



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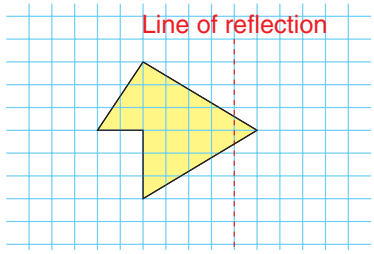
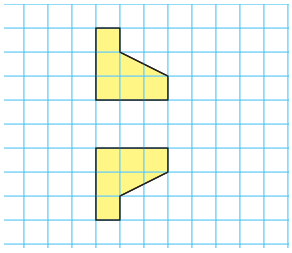
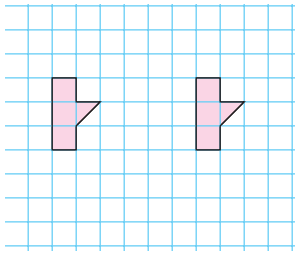


2. Which pictures show a reflection? How do you know?
Describe where the line of reflection is.

a)



b)



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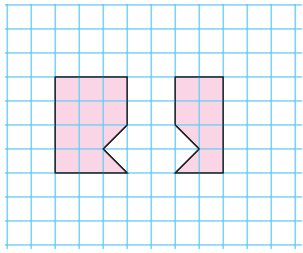
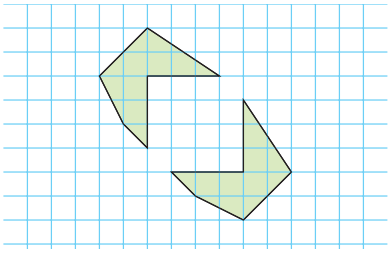


3. In question 2, do any pictures show a translation?
If so, describe the translation.

4. Each picture shows a shape and its reflection image.

a)



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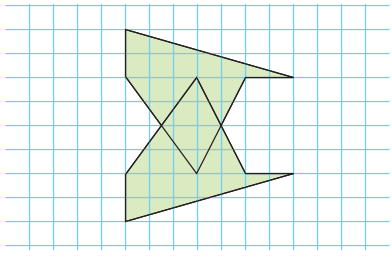
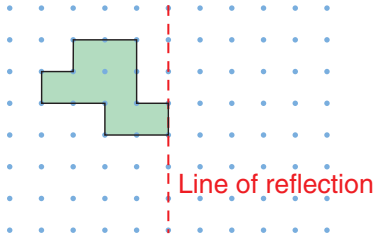


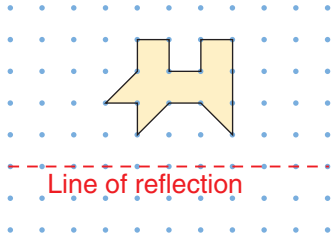
Copy each picture on grid paper.
Draw the line of reflection. Explain how you did this.
How do you know the line of reflection is drawn correctly?

5. Copy each shape and line of reflection on dot paper.
Predict where each reflection image will be.
Draw each image to check your prediction.

a)



b)



6. Print the letters of the alphabet as capital letters.

a) Draw a horizontal line above each letter.
Place a Mira on the line.
Which letters look the same in the Mira?

b) Draw a vertical line beside each letter.
Place a Mira on the line.
Which letters look the same in the Mira?

c) Create three words whose images read the same as the words when a Mira is placed above the letters.

7. Draw a shape on dot paper.
Draw and label a line of reflection.
Draw the image of the shape in the line of reflection.

a) Use a ruler.
Join two matching points on the shape and its image.

b) Use a ruler.
Measure the distance between each point and the line of reflection.
What do you notice?

c) What do you notice about the angle between the line you drew in part a and the line of reflection?

Show your work. Explain your thinking.

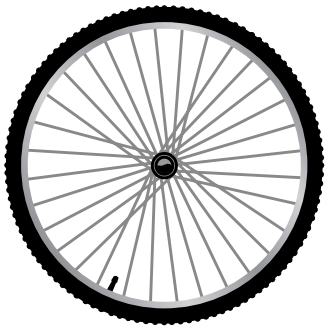
Reflect

How are a translation and a reflection alike?
Draw a shape and its image that could show both a reflection and a translation.

LESSON 4

Rotations

A bicycle wheel turns about the centre of the wheel.



What other examples are there of things that turn? Explain how they turn.

Explore

You will need several pieces of paper, tracing paper, a ruler, a compass, and scissors.

- Use a ruler.
  Draw a shape with straight sides in the centre of a piece of paper.
- Use tracing paper.
  Draw an identical shape on another piece of paper.
  Cut out this shape.
  Place it on top of the first shape you drew.
- Put your compass point at one vertex.
  Turn the shape to a new position.
  Draw the shape in its new position.
  Label this shape Image A.
- Return your shape to its original position.
  Turn the shape in the opposite direction.
  Draw the shape in its new position.
  Label this shape Image B.
- In each case, how do the positions of the original shape and its image compare?

Show *and* Share

Compare your picture and ideas with another pair of classmates. Did you have the same ideas about how a shape compares with its image after a rotation? Explain.

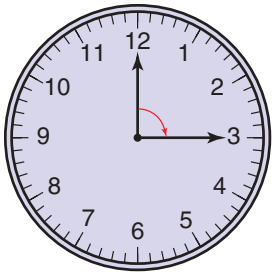
Connect

When a shape turns about a point, it is *rotated* from one position to another.

The movement is a **rotation**, or *turn*. When we draw the shape in its new position, we draw a **rotation image** of the shape.

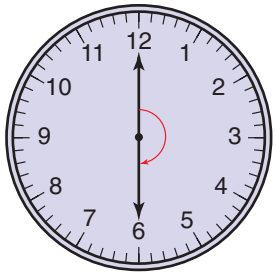
After 1 complete turn, a shape is back to where it started.

When the minute hand on a clock moves from 12 to 3, it moves a **quarter turn**.



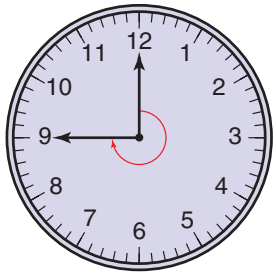
$\frac{1}{4}$ turn

When the minute hand moves from 12 to 6, it moves a **half turn**.



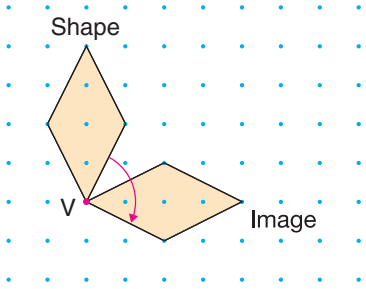
$\frac{1}{2}$ turn

When the minute hand moves from 12 to 9, it moves a **three-quarter turn**.

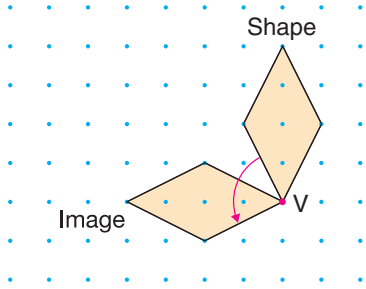


$\frac{3}{4}$ turn

A shape can rotate **clockwise** about a vertex V:

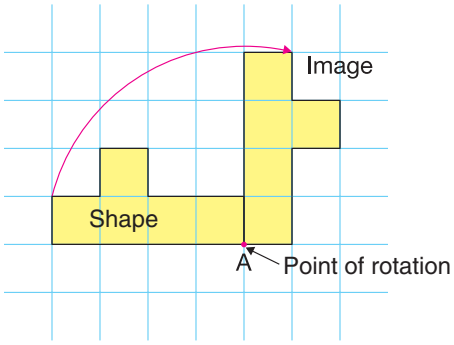


A shape can rotate **counterclockwise** about a vertex V:

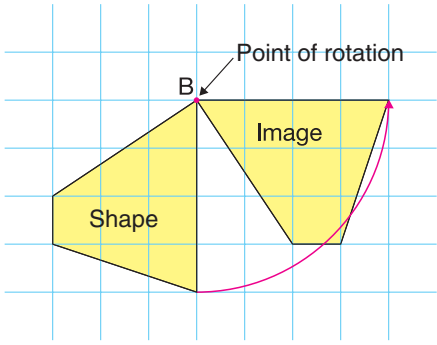


Any turn less than 1 complete turn is a fraction of a turn clockwise or counterclockwise.

This shape has rotated a $\frac{1}{4}$ turn clockwise, about vertex A.
This point is called the **point of rotation**.



This shape has rotated a $\frac{1}{4}$ turn counterclockwise, about vertex B.



A rotation is described by:

- the direction of the turn (clockwise or counterclockwise),
- the fraction of the turn, and
- the point of rotation

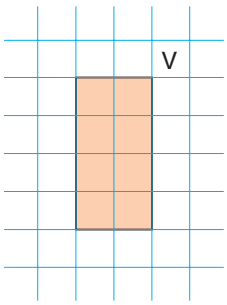
A shape and its rotation image have different orientations. The shape and its image face different ways for any rotation that is less than 1 complete turn.

A reflection, a rotation, and a translation are **transformations**.

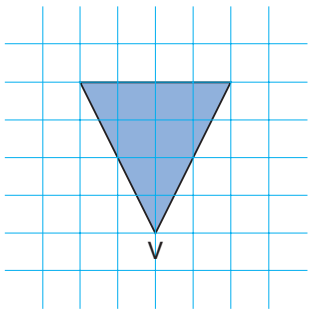
Practice

1. Copy each shape below on grid paper. For each shape:
- Rotate the shape about vertex V, using the rotation given.
- Draw the rotation image.
- Describe the position and orientation of the image.

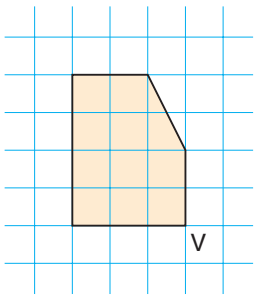
a) a $\frac{1}{4}$ turn counterclockwise



b) a $\frac{1}{2}$ turn clockwise

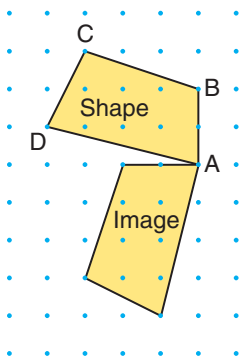


c) a $\frac{3}{4}$ turn clockwise

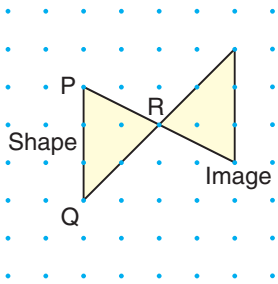


2. Each picture below shows a shape and its rotation image.
Describe the rotation. Include the direction of the turn.

a)

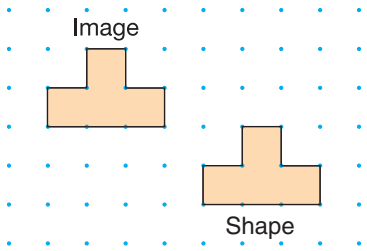


b)

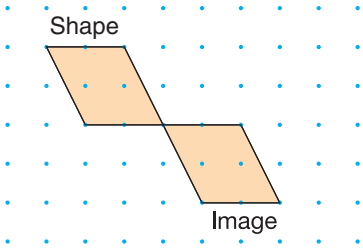


3. Which pictures show a rotation?
How do you know? Describe the rotation.

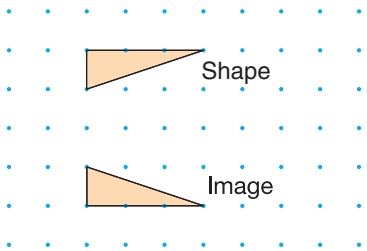
a)



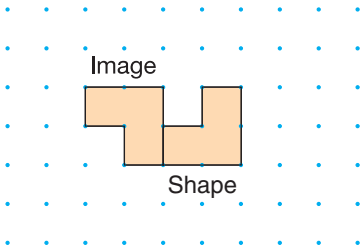
b)



c)



d)

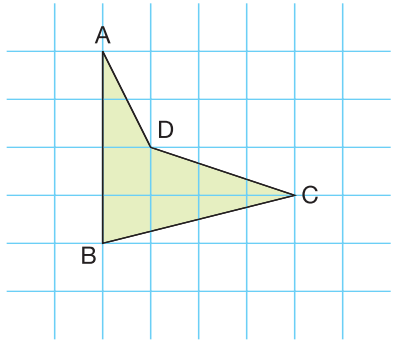


4. Did any of the pictures in question 3 show a translation? A reflection?
If so, identify the picture and describe the transformation.

5. Copy this shape.
Trace the shape on tracing paper.
Use the tracing to rotate the shape.
Predict the position of the image
after each rotation below.
Draw each image to check your prediction.

a) a $\frac{1}{4}$ turn clockwise about vertex A

b) a $\frac{3}{4}$ turn counterclockwise about vertex A

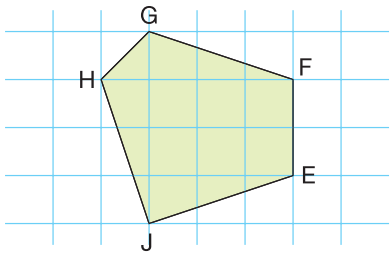


6. Copy this shape.
Use tracing paper to rotate the shape:

a) a $\frac{1}{2}$ turn clockwise about vertex E

b) a $\frac{1}{2}$ turn counterclockwise about vertex E

What do you notice about the rotation images?



7. Copy this shape on grid paper.

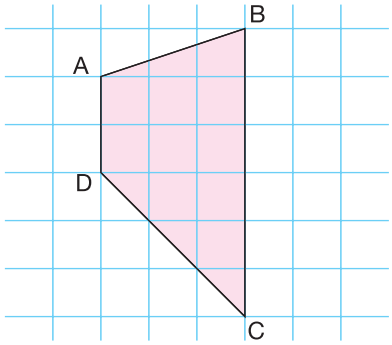
a) Rotate the shape about a vertex. Describe the direction of the turn, the fraction of the turn, and the point of rotation. Draw the image.

b) Repeat part a for a different direction.

c) Repeat part a for a different fraction.

d) Repeat part a for a different point of rotation.

Show your work.



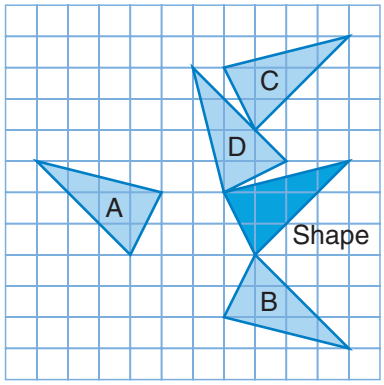
8. Describe the transformation that moves the shape to each image.
Can you describe any movements in more than one way? Explain.

a) Image A

b) Image B

c) Image C

d) Image D



At Home

Look for an example of each transformation.
Describe each transformation and explain how you identified it.

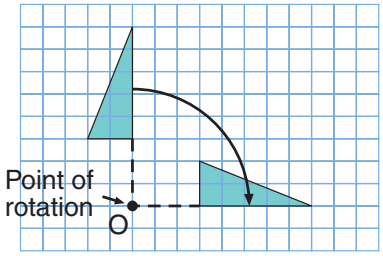
Reflect

When you see a shape and its image, how do you know if they show a reflection, a rotation, or a translation?
Use diagrams to explain.

LESSON 5

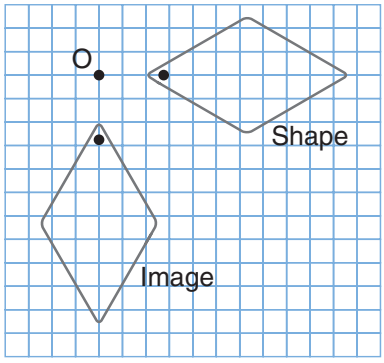
Exploring Different Points of Rotation

A shape can rotate about a point of rotation that is not on the shape.

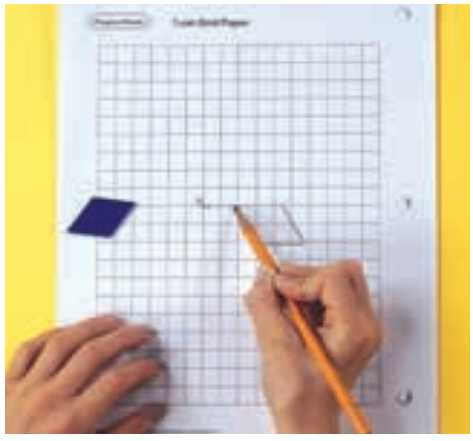


Explore

You will need grid paper, Pattern Blocks, and a ruler.



➤ A blue Pattern Block was placed on grid paper and traced.
The block was rotated about point O and traced again.
Describe different ways the block could have moved.
Tell about the fraction of the turn, the direction, and the point of rotation.
How do the shape and its image compare?



➤ Trace the blue Pattern Block on grid paper as shown.
Extend one side and mark this endpoint O.
Use point O as the point of rotation.
Choose clockwise or counterclockwise.
Rotate the block a $\frac{1}{2}$ turn about point O.
Trace its new position.

Show *and* Share

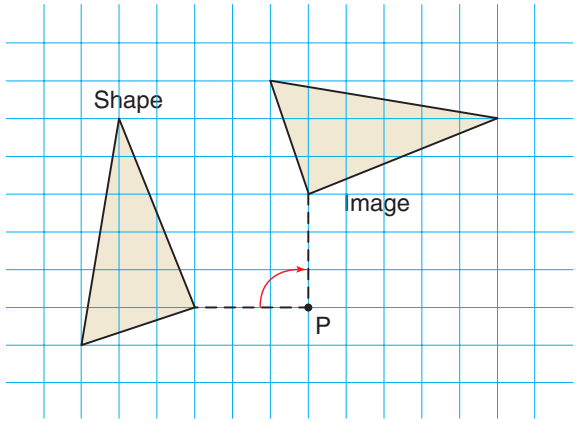
Exchange your tracings with a pair of students who rotated their block in the direction opposite to yours.
What do you notice? Explain.

Lesson Focus | Rotate a shape about a point of rotation not on the shape.

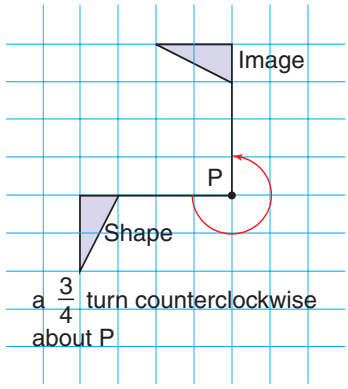
Connect

➤ We can use tracing paper to find the image when we rotate a shape.

- Place the tracing paper so the bottom right corner is on point P.
- Trace the shape.
- Hold the tracing paper in place with your pencil at point P.
  Rotate the tracing paper a $\frac{1}{4}$ turn clockwise.
- Note the position of the image of the shape.
- Lift the tracing paper and draw the image in place.
  Label the image.



➤ We can predict the position of the image formed when we rotate a shape.
Visualize the shape as a flag whose pole joins any vertex to the point of rotation.
The pole rotates, but its length does not change.

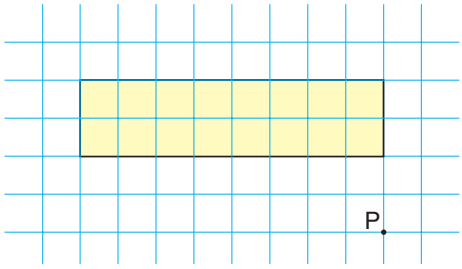


Practice

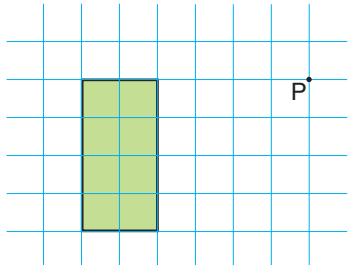
Use tracing paper when needed.

1. Copy each rectangle and point P on grid paper.
Draw each image after a $\frac{1}{4}$ turn clockwise about point P.

a)

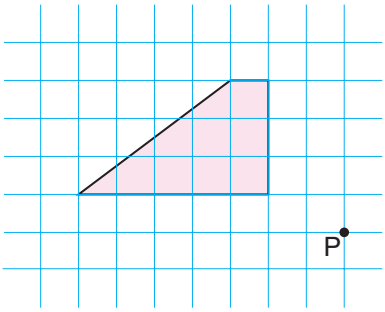


b)

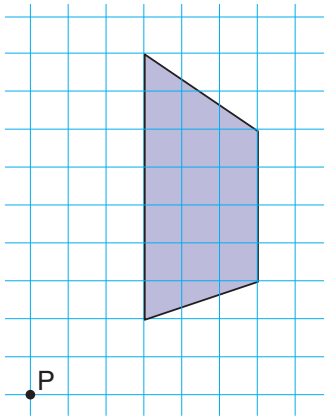


2. Copy each trapezoid and point P on grid paper.
Draw each image after a $\frac{1}{2}$ turn clockwise about P.

a)



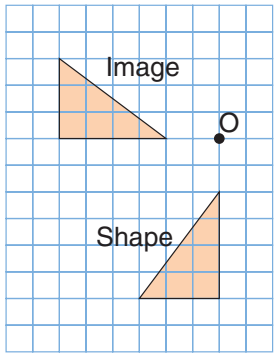
b)



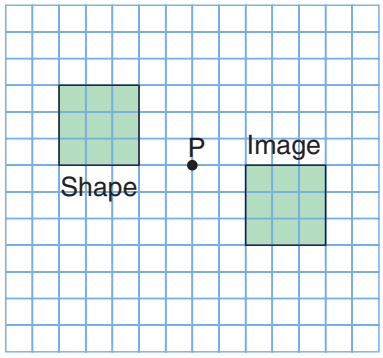
3. Describe each rotation.
Include:

- the fraction of the turn
- the point of rotation
- the direction

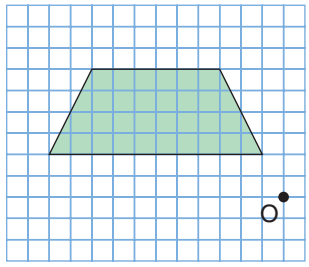
a)



b)



4. Copy this trapezoid and point O on grid paper.



a) Draw the image after a $\frac{1}{4}$ turn clockwise about point O.

b) Draw the image after a $\frac{1}{2}$ turn counterclockwise about point O.

c) How can you tell if you have drawn the correct images?

5. Draw a quadrilateral on grid paper.
Choose a point outside the quadrilateral.
Rotate the quadrilateral about the point you chose.
Draw its rotation image.
Describe the rotation.

Reflect

When you see a shape and its rotation image, how can you tell if the point of rotation is on or off the shape?

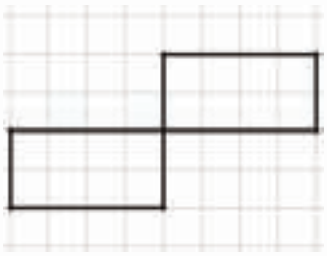
Transformations on a Computer

Use dynamic geometry software.
Open a new sketch.
Check that the distance units are centimetres.
Display a grid.

If you need help at any time, use the Help menu.

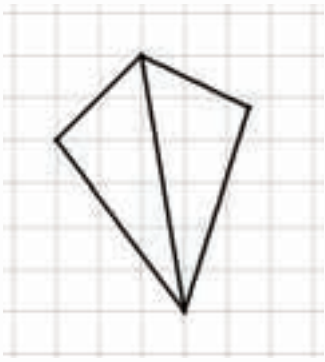
Translating a Shape

Construct a rectangle. Select the rectangle.
Translate the rectangle 4 squares left and 2 squares down.
Print the rectangle and its translation image.



Reflecting a Shape

Construct a triangle.
Select one side of the triangle as the line of reflection.
Select the triangle. Reflect it in the line of reflection.
Print the triangle and its reflection image.



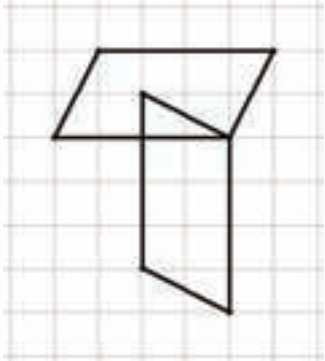
Rotating a Shape

Construct a parallelogram.
Select a vertex of the parallelogram as the point of rotation.
Select the parallelogram. Rotate it a $\frac{1}{4}$ turn counterclockwise.
Print the parallelogram and its rotation image.

Identifying Rotations

Work with a partner. Take turns.

➤ Construct a shape on the grid.
Choose a rotation and construct the rotation image.
Print the picture.
Have your partner identify the rotation.
Remind her to include:
- the point of rotation
- the fraction of the turn
- the direction of the turn



➤ Repeat the steps above for different points of rotation, different fractions of a turn, and different directions.

Predicting the Image

Work with a partner.
Take turns.

➤ Construct a shape on the grid.
Print the shape.
Choose a translation.
Tell your partner what it is.
Have your partner predict where the translation image will be.
Translate the shape and draw its image.
Print the shape and its image to verify your partner's prediction.

➤ Repeat the steps above for a reflection.

➤ Repeat the steps above for a rotation.

Identifying a Transformation

Work with a partner.
Take turns.

➤ Choose a transformation.
Construct a shape and its image.
Have your partner look at the screen and identify the transformation.

➤ Repeat the steps above for different transformations and different shapes.

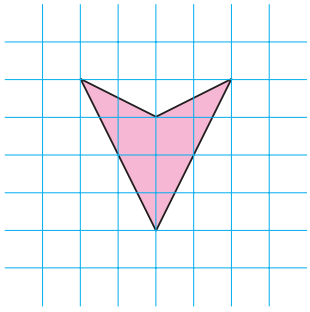
Reflect

How does each shape and its image compare?
Do the comparisons match those you made from pictures you drew in earlier lessons? Explain your ideas.

Unit 8 Show What You Know

LESSON
1
3
4

1. Copy the shape on grid paper.



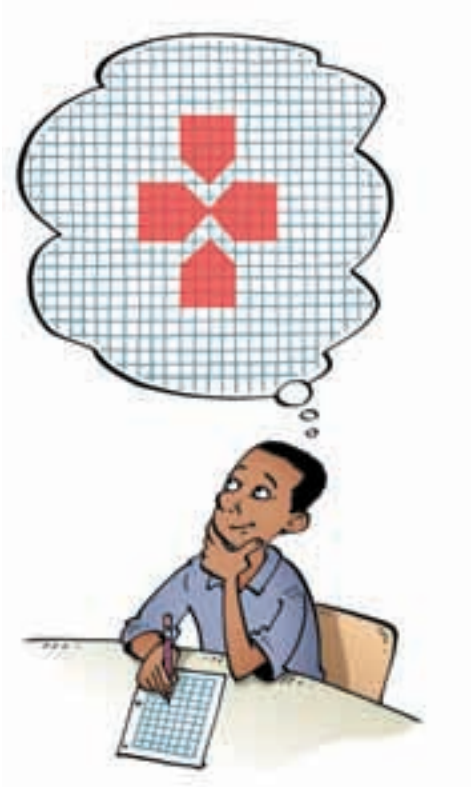
a) Translate the shape in any direction you like.
Draw its translation image.

b) Draw a line of reflection.
Draw the reflection image.

c) Choose a point of rotation and a fraction of a turn.
Rotate the shape and draw its rotation image.

d) Describe the position and orientation of each image in parts a, b, and c.
How does each description help you identify the transformation?

2. Draw a shape on grid paper.

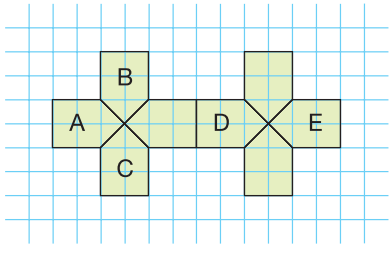


a) Translate the shape any way you like.
Draw its translation image.
Record the translation.
Include each direction and the number of squares moved.

b) Reflect the shape.
Draw its reflection image.
Label the line of reflection.
Find how far the shape and its image are from this line.

c) Rotate the shape.
Draw its rotation image.
Describe the rotation.
Include the direction of the turn, the fraction of the turn, and the point of rotation.

3. Describe a transformation that would move shape A to each image.



a) Image B **b)** Image C
c) Image D **d)** Image E

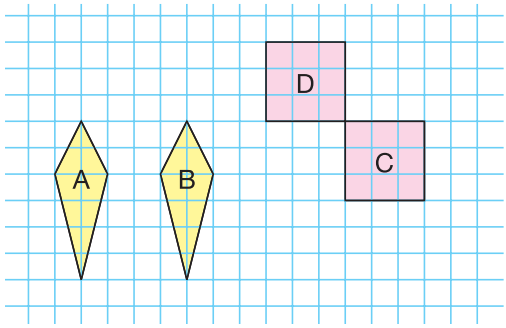
LESSON

1

4. Describe the translation that moves:

a) Shape B to Image A

b) Shape D to Image C



1 3 4

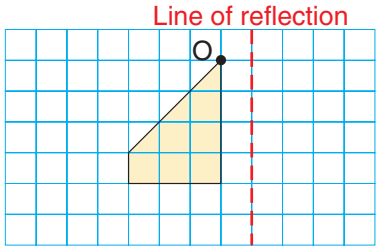
5. In question 4, which other transformation would move each shape to its image?

6. Copy this shape on grid paper.
Predict the position of the image after each transformation below.
Draw each image to check your prediction.

a) a reflection in the line of reflection

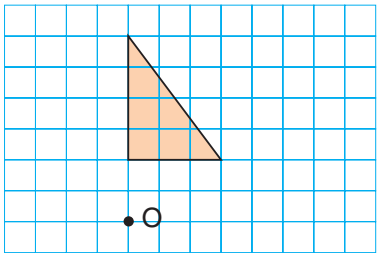
b) a translation 3 squares right and 4 squares up

c) a $\frac{1}{4}$ turn counterclockwise about O



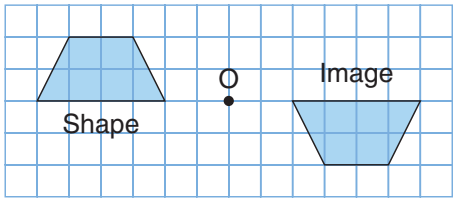
5

7. Copy this triangle and point O on grid paper.
Draw the image after a $\frac{1}{4}$ turn clockwise about O.



1 3 4 5

8. Describe the transformation that moves the shape to its image.



UNIT 8

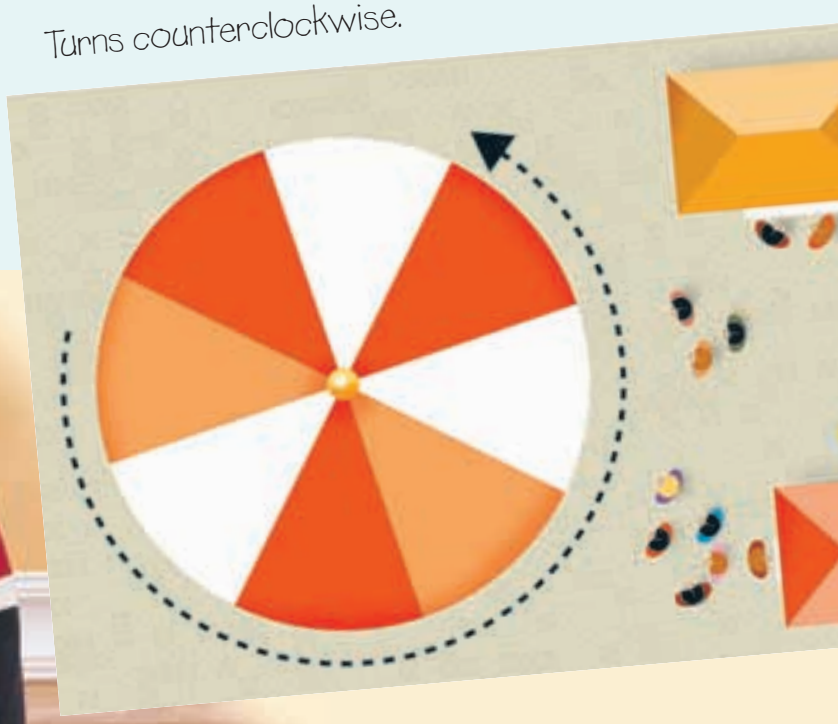
Learning Goals

- ☑ translate, reflect, and rotate a shape
- ☑ draw and describe images after transformations
- ☑ identify a transformation

Unit Problem

At the Amusement Park

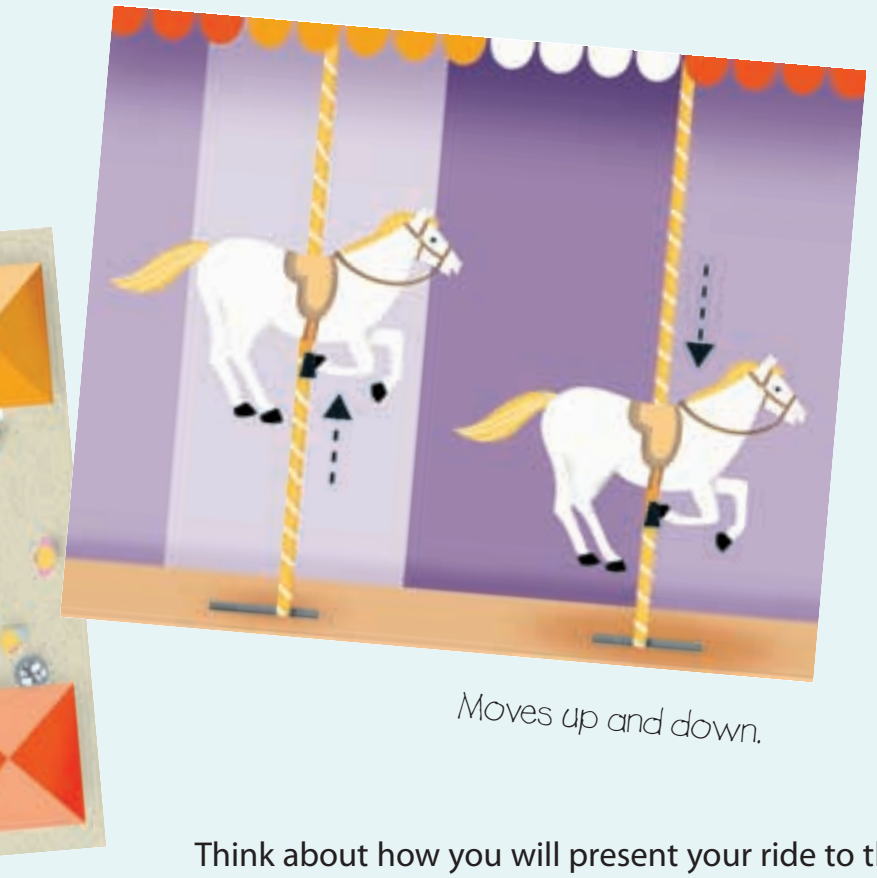
Design a ride for an amusement park.
The ride must move people
in at least 2 *different* ways.



Check List

Your work should show

- ☑ how you designed a ride
- ☑ a clear presentation of your ride
- ☑ an explanation of how the ride moves
- ☑ how you used geometric language to describe your work



Moves up and down.

Think about how you will present your ride to the class.

Will you
– make a drawing?
– make a model?
– write about it?
– talk about it?

How does your ride move people in at least 2 different ways?

Reflect on Your Learning

What did you learn about how shapes can move?
Use words and pictures to explain.

Investigation

Dinomaze

Part 1

Dinosaurs were first discovered in England in 1824.
Since then, many dinosaur fossils have been found in Western Canada.
Some dinosaurs, such as the Edmontosaurus and the Columbosauripus have been named after a Canadian city or province.

- Use books, magazines, or the Internet to learn which dinosaurs have been found in each province or territory of Canada.
  Record your findings in a table, chart, or on a map.

Great care is taken when fossils are excavated to avoid breaking the remains. The location of a fossil find is important, so a region is often searched systematically, using a grid.

Part 2

Play *Dinomaze* with a partner.
This game uses dinosaurs as obstacles, and number cubes to determine a translation.
You will need a copy of the game board, a red number cube and a green number cube, and 2 different coloured counters.

Rules:

- Each player rolls a number cube.
  The player who rolls the greater number starts.
- Take turns to roll both cubes.
  The green cube tells how many squares to move right or left.
  The red cube tells how many squares to move up or down.
- You must always move horizontally first, and then vertically.
- A dinosaur square is an obstacle.
  You cannot cross a dinosaur square, or land on it.
- If you cannot move in one direction, you must move in the opposite direction if it is possible. If you cannot move, you miss that turn.
- Both players cannot be on the same square at the same time.
  The only exception to this is the START square.
- The winner is the first player to land on the END square.

Play the game several times.

- Which strategies did you use to help you win?
- Which series of translations would take you to the END in 6 turns or fewer?
- Which series of translations would take you to the END in the fewest turns?

Take It Further

Use grid paper.
Design your own *Dinomaze* game.
You may choose a different shaped game board, or use spinners instead of number cubes.