UNIT 2

Understanding

At the Apiary

Learning Goals

- use place value to represent whole numbers greater than one million
- solve problems involving large numbers, using technology
- determine multiples and factors of numbers less than 100
- solve problems involving multiples
- identify composite and prime numbers
- apply the order of operations to solve multi-step problems, with or without technology
- demonstrate an understanding of integers

Number

Honeybees have been producing honey for more than 150 million years. Honeybees gather nectar from flowers. They convert the nectar to honey and store it as food in the beehive. A colony of honeybees produces more honey than it needs. For 6000 years, beekeepers have harvested honey for people to eat.

- Lesley has 20 hives.
  Each hive has about 75 000 honeybees.
  How could you find out about how many honeybees Lesley has?
- A honeybee travels about 195 km in 50 round trips to collect enough nectar to make 1 g of honey.
  About what distance does a honeybee travel in one round trip? How do you know?
- What else do you know about Lesley's honeybees?

Key Words

- billion
- trillion
- common multiples
- prime number
- composite number
- common factors
- order of operations
- expression
- integer
- positive integer
- negative integer
- opposite integers

LESSON 1

Exploring Large Numbers

The world's all-time best-selling copyright book is *Guinness World Records*. From October 1955 to June 2002, 94 767 083 copies were sold.

Suppose the number is written in this place-value chart. Where will the digits 9 and 4 appear?

Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones

Explore

Here are some of the world records reported in the *Guinness World Records 2008*.

- The largest bag of cookies was made in Veenendaal, Netherlands. It contained 207 860 cookies.
- The greatest attendance at an Olympic Games was 5 797 923, in Los Angeles in 1984.
- The most dominoes toppled by a group was 4 079 381, out of a possible 4 400 000. This took place at Domino Day 2006 in Leeuwarden, Netherlands.
- The longest gum-wrapper chain contains 1 192 492 wrappers. The maker of the chain has been working on it since 1965.

➤ Take turns reading the records aloud.

➤ Each of you chooses 2 numbers from the records. Show each number in as many ways as you can.

Lesson Focus | Read and write whole numbers up to and greater than one million.

Show *and* Share

Share your work with another pair of students.
Talk about the different ways you showed your numbers.

Connect

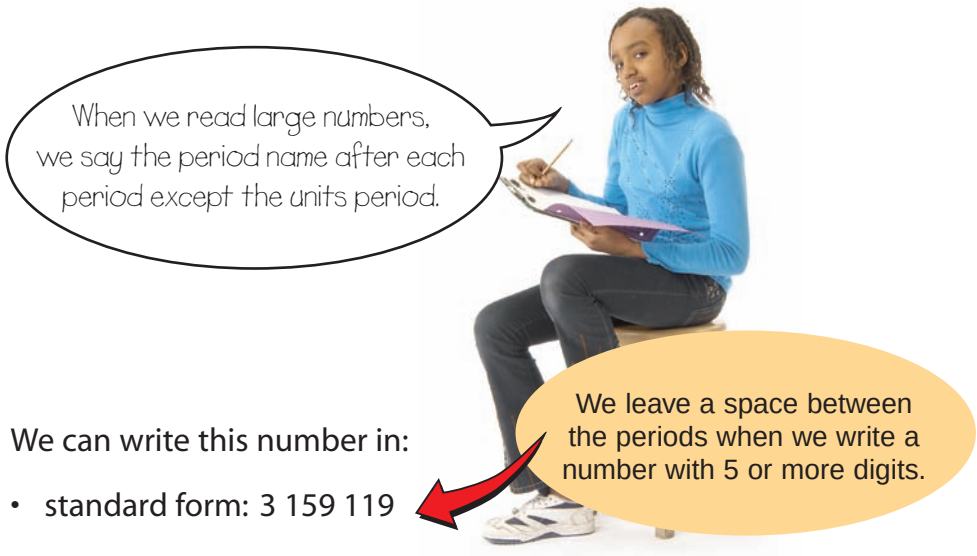
➤ These patterns in the place-value system may help you read and write large whole numbers.

- From right to left, each group of 3 place values is called a *period*.
- Within each period, the digits of a number are read as hundreds, tens, and ones.
- Each position represents ten times as many as the position to its right.
  For example, 2 hundreds = 20 tens and 4 ten thousands = 40 thousands

This place-value chart shows the number of items in the world's largest collection of matchbook covers, 3 159 119.

Millions Period			Thousands Period			Units Period		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
		3	1	5	9	1	1	9
		3 000 000	100 000	50 000	9000	100	10	9

We read this number as:
three *million* one hundred fifty-nine *thousand* one hundred nineteen



We can write this number in:

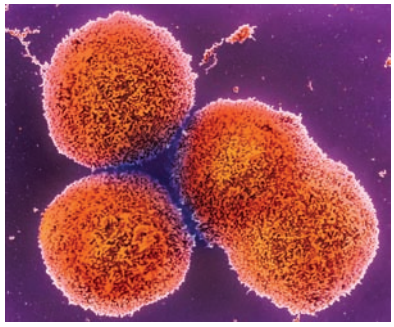
- standard form: 3 159 119
- expanded form: 3 000 000 + 100 000 + 50 000 + 9000 + 100 + 10 + 9
- number-word form: 3 million 159 thousand 119

➤ The place-value chart can be extended to the left to show greater whole numbers. This place-value chart shows the approximate number of cells in the human body.

One thousand million is one **billion**.
One thousand billion is one **trillion**.

Trillions			Billions			Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O	H	T	O	H	T	O
	5	0	0	0	0	1	0	0	0	0	0	0	0	0

We write: 50 000 100 000 000
We say: fifty trillion one hundred million



Practice

1. Write each number in standard form.
a) 20 000 000 + 4 000 000 + 300 000 + 40 000 + 2000 + 500 + 80 + 4
b) 6 million 276 thousand 89
c) two billion four hundred sixty million sixty-nine thousand eighteen

2. How many times as great as one thousand is one million?
Use a calculator to check your answer.

3. Write each number in expanded form.
a) 75 308 403 **b)** 64 308 470 204 **c)** 99 300 327

4. Write the value of each underlined digit.
a) 6<u>2</u>7 384 **b)** 5<u>4</u> 286 473 **c)** 41 <u>9</u>62 014 **d)** 2<u>5</u> 041 304 000

5. Write the number that is:
a) 10 000 more than 881 462
b) 100 000 less than 2 183 486
c) 1 000 000 more than 746 000
d) one million less than 624 327 207
Tell how you know.

6. China is the most populated country in the world. In 2007, it had an estimated population of one billion three hundred twenty-one million eight hundred fifty-one thousand eight hundred eighty-eight.
Write this number in standard form and in expanded form.

7. The largest known prehistoric insect is a species of dragonfly.
It lived about 280 000 000 years ago.
Write this number in words.

8. North America's largest shopping centre is in Edmonton, Alberta.
It covers an area of 492 386 m^2 and cost about $1 200 000 000 to build.
Write these numbers in a place-value chart.

9. A student read 3 000 146 as "three thousand one hundred forty-six."
How would you explain the student's error?

10. I am a number between 7 000 000 and 8 000 000.
All my digits are odd.
All the digits in my thousands period are the same.
All the digits in my units period are the same.
The sum of all my digits is 31.
What number am I?
Give as many answers as you can.
What strategies did you use to find the mystery number?

11. In November, 2005, six-year-old Brianna Hunt found a fossilized squid near her home on the Blood Reserve, outside Lethbridge, Alberta.
The fossil is believed to be about 73 million years old.
Write this number in standard form.

12. The table shows estimates of the populations of some cities in 2015.
Order the cities from least to greatest expected population.

City	Expected Population in 2015
Dhaka (Bangladesh)	22 766 000
Mumbai (India)	22 577 000
Tokyo (Japan)	27 190 000

13. Explain the meaning of each 6 in the number 763 465 284 132.

14. Describe three examples where large numbers are used outside the classroom.

15. How do the patterns in the place-value system help you to read and write large numbers?

Math Link

Number Sense

A googol is a number represented by 1 followed by 100 zeros.
The word *googol* was created in 1920 by an American mathematician, Edward Kasner, on the basis of his 9-year-old nephew's word for a very large number.

Reflect

What patterns are there in a place-value chart? How can you use these patterns to read a number such as 5 487 302?

At Home

Search the Internet, or look through magazines.
Find examples where large numbers are used.
How are the numbers written?

LESSON 2

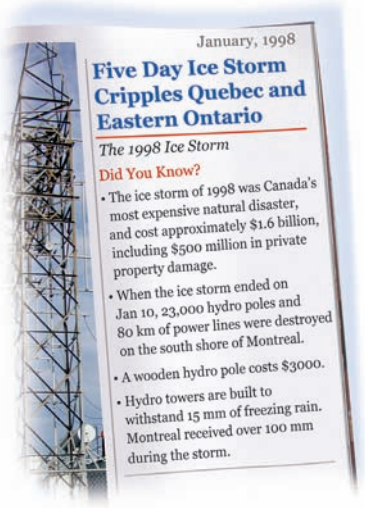
Numbers All Around Us

We add, subtract, multiply, or divide with numbers to solve problems.
Addition, subtraction, multiplication, and division are *operations*.
We use numbers to understand and describe our world.

Saturday, August 23, 2003

Southern BC Fires Approach City

KELOWNA, BC – Police with bullhorns ordered a staggering 20,000 Kelowna residents to evacuate their homes as fast-moving wildfires moved closer.
Thick smoke choked Kelowna's 100,000 residents and at times clouded the view of nearby Okanagan Lake.
The fire prompted the closure of Route 97, between Okanagan Falls and Penticton, and brought the total number of evacuees driven out by the fire to 30,000.
More than 3,000 firefighters from British Columbia and elsewhere in Canada are battling over 825 fires in the province.



January, 1998

Five Day Ice Storm Cripples Quebec and Eastern Ontario

The 1998 Ice Storm

Did You Know?

- The ice storm of 1998 was Canada's most expensive natural disaster, and cost approximately $1.6 billion, including $500 million in private property damage.
- When the ice storm ended on Jan 10, 23,000 hydro poles and 80 km of power lines were destroyed on the south shore of Montreal.
- A wooden hydro pole costs $3000.
- Hydro towers are built to withstand 15 mm of freezing rain. Montreal received over 100 mm during the storm.

June, 2002

Canadian Prairies Prepare For Grasshopper Plague

Reuters--Winnipeg, MB. – Western Canadian farmers are readying for battle against swarms of voracious grasshoppers that are poised to destroy millions of dollars worth of crops.
The government of Alberta recently announced a $10.3 million pre-emptive strike against the grasshoppers. If left uncontrolled, the province said grasshopper damage could cost between $80 million and $100 million this year.

Explore

Read the articles above.

➤ Use the numbers in the articles.
Write a problem you would solve using each operation:
- addition
- subtraction
- multiplication
- division

➤ Estimate first. Then solve your problems.
Use a calculator when you need to.

➤ Trade problems with another pair of students.
Solve your classmates' problems.

Show *and* Share

How did you decide which operation to use to solve each of your classmates' problems?
How did you decide whether to use a calculator?
How do you know your answers are reasonable?

Connect

The population of Canada was about 32 980 000 in July 2007.
Data show that there were about 497 cellular phones per 1000 people in that year.
How many cellular phones were there in Canada in 2007?

This is a 2-step problem.

➤ First, find how many groups of 1000 there are in 32 980 000.
To find how many equal groups, divide: $32\,980\,000 \div 1000 = 32\,980$

➤ There are about 497 cellular phones for one group of 1000.
To find how many cellular phones for 32 980 groups of 1000, multiply:
$32\,980 \times 497 = 16\,391\,060$

There were about 16 391 060 cellular phones in Canada in 2007.

Estimate to check the answer is reasonable.
Use benchmarks:
- 32 980 000 is closer to 30 000 000 than to 40 000 000.
  $30\,000\,000 \div 1000 = 30\,000$
- 497 is closer to 500 than to 400.
  $30\,000 \times 500 = 15\,000\,000$

16 391 060 is close to 15 000 000.
So, 16 391 060 is a reasonable answer.

Practice

Use a calculator when you need to.

1. The ticket agent sold 357 adult tickets and 662 student tickets for a concert. How much money did the ticket agent take in? Explain how you know your answer is reasonable.

2. The table shows the populations of the western provinces and territories in 2006.

Provinces and Territories	Population
British Columbia	4 113 487
Alberta	3 290 350
Saskatchewan	968 157
Manitoba	1 148 401
Yukon Territory	30 372
Northwest Territories	41 464
Nunavut	29 474

a) Find the total population of the 4 western provinces.

b) How many more people live in Saskatchewan than in Nunavut?

c) Make up your own problem about these data. Solve it.

3. The total population of Canada was 30 007 094 in 2001 and 31 612 897 in 2006. By how much did the population increase from 2001 to 2006?

4. Monarch butterflies migrate from Canada to Mexico every fall. It is estimated that the butterfly travels about 82 km each day. Suppose the butterfly travels from Edmonton to El Rosario. This is a distance of about 3936 km.
How many days does it take?
How did you decide which operation to use?

5. The Fairview High School community of 1854 students and 58 teachers attended a special performance of a play at a local theatre.
The theatre has 49 rows, with 48 seats in each row.

a) Were any seats empty? How do you know?

b) If your answer to part a is yes, find the number of empty seats.

6. This table shows the number of participants at the 2002 and 2006 North American Indigenous Games.

Year	Athletes	Coaches, Managers, and Chaperones
2002 (Winnipeg)	6136	1233
2006 (Denver)	7415	1360

a) What was the total number of participants in 2002?

b) How many more athletes participated in 2006 than in 2002?

c) About how many times as many athletes participated in 2002 as coaches, managers, and chaperones?

How did you decide which operation to use each time?

Opening Ceremonies, 2002 North American Indigenous Games, Winnipeg

7. The food bank received 325 cases of 24 cans of soup, and 227 cases of 48 cans of soup.
Estimate first. Then find how many cases of 12 cans of soup can be made.

8. Ms. Talby's hens laid 257 dozen eggs last month.
   a) About how many eggs is that?
   Explain your estimation strategy.
   b) Exactly how many eggs is that?
   How do you know your answer is reasonable?

9. The owner of a building renovated 18 apartments.
Painting cost $5580 and new lights cost $3186.
   a) Which operation or operations will you use to find the cost for each apartment? Explain.
   b) Estimate this cost. Explain the strategy you used.
   c) Find the exact cost.

10. A newspaper prints 8762 papers, each with 16 pages.
A roll of newsprint can be used to print 6150 pages.
About how many rolls of newsprint are required?
Show your work.
How do you know your answer is reasonable?

11. The world's longest novel, *À la recherche du temps perdu* by Marcel Proust of France, contains about 9 609 000 letters.
   a) Suppose each page contains about 2400 letters.
   About how many pages does the novel contain?
   b) Suppose it took Jacques 85 days to read the novel.
   He read the same number of pages per day.
   About how many pages did Jacques read each day?
How do you know your answers are reasonable?

Reflect

When you read a problem, how do you decide which operation you need to use to solve the problem?
Use examples from this lesson to explain your answer.

LESSON 3

Exploring Multiples

Explore

On Thursday morning, the local radio station held a call-in contest.

- Every third caller won a T-shirt.
- Every seventh caller won a baseball cap.

In 50 calls, which callers won a T-shirt? A baseball cap? Both prizes?

Use any materials you like to solve this problem.
Show how you used materials to solve this problem.

Show *and* Share

Share your answers with another pair of students.
What strategies did you use to solve the problem?
Discuss how using materials helped.
Describe any patterns you noticed.

Connect

To find the multiples of a number, start at that number and count on by the number.
You can use a hundred chart to find the multiples of a number.

The multiples of 4 are:
4, 8, 12, 16, 20, 24, 28, 32, 36, 40, …

The multiples of 6 are:
6, 12, 18, 24, 30, 36, …

12, 24, and 36 appear in both lists.
They are multiples of 4 and of 6.
They are **common multiples** of 4 and 6.
12 is the *least common multiple* of 4 and 6.

Each common multiple of 4 and 6 is divisible by 4 and by 6.

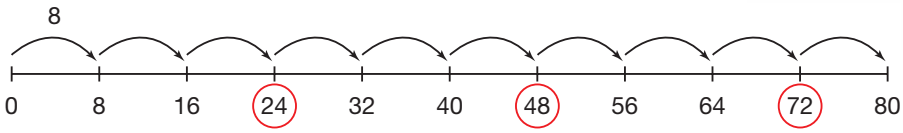
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

The least common multiple is the first common multiple.

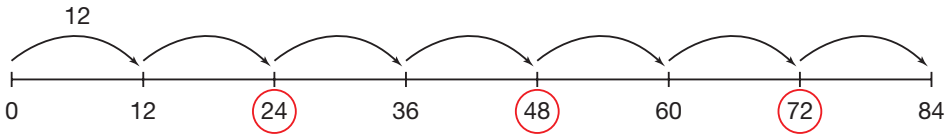
We can use multiples to solve some problems.

Wieners are sold in packages of 12.
Hot dog buns are sold in packages of 8.
Suppose you plan to sell about 75 hot dogs to raise money for charity.
You do not want any wieners or buns left over.
How many packages of each should you buy?

You can use number lines to find the multiples of 8 and 12.
To find the multiples of 8, start at 0 and skip count by 8.



To find the multiples of 12, start at 0 and skip count by 12.



Circle the common multiples: 24, 48, 72
Since 72 is close to 75, you should buy 72 wieners and 72 buns.
You skip counted by eight 9 times to reach 72, so buy 9 packages of buns.
You skip counted by twelve 6 times to reach 72, so buy 6 packages of wieners.

Practice

You may use a hundred chart or number lines to model your solutions.

1. List the first 10 multiples of each number.
a) 2 **b)** 5 **c)** 8 **d)** 7

2. List the first 6 multiples of each number.
a) 12 **b)** 11 **c)** 16 **d)** 15

3. Which numbers below are multiples of 6?
What strategy did you use to find out?
36 70 66 42 54 27 120 81

4. Which of the numbers 21, 24, 45, 30, 42, 60, and 84 are multiples of:
a) 3? **b)** 12? **c)** 7? **d)** 15?

5. Find the first 3 common multiples of each pair of numbers.

a) 4 and 5 **b)** 7 and 4 **c)** 3 and 9 **d)** 10 and 15

6. Find the first 3 common multiples of each set of numbers.
Which is the least common multiple? Explain your work.

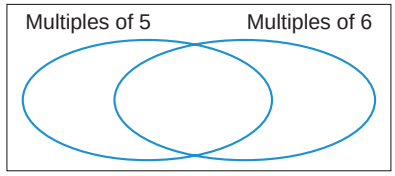
a) 3, 4, and 6 **b)** 2, 3, and 4 **c)** 4, 5, and 10

7. Find all the common multiples of 8 and 9 that are less than 100.

8. Two TV movies start at 8:00 P.M.
One channel airs commercials every 6 min.
The other channel airs commercials every 9 min.
When will the two channels start commercial breaks at the same time?

9. A spider has 8 legs. An ant has 6 legs.
There are a group of spiders and a group of ants.
The groups have equal numbers of legs.
What is the least number of spiders and ants in each group? Show your work.

10. Make a large copy of this Venn diagram.
Sort these numbers.
45, 24, 52, 30, 66, 15, 85, 90, 72, 60, 20, 38
What can you say about the numbers in the overlap?



11. Taho plays shinny every 2 days. He plays lacrosse every 3 days.
Suppose Taho plays shinny and plays lacrosse on October 1.
What are the next 3 dates on which he will play shinny and play lacrosse? Explain how you know.

12. Find the first 2 common multiples of 36 and 48.

13. Which numbers are common multiples of 8 and 3?
How did you find out?

a) 32 **b)** 72 **c)** 48
d) 54 **e)** 66 **f)** 96

14. Veggie patties are sold in packages of 5.
Buns are sold in packages of 8.
You need about 125 veggie burgers for a school barbecue.
You do not want any patties or buns left over.
How many packages of each should you buy?
What strategy did you use to find out?

15. Kevin, Yone, and Miroki work part-time at the YMCA in Kamloops.
Kevin works every second day.
Yone works every third day.
Miroki works every fourth day.
Today, they worked together.
When will they work together again?
Explain how you know.

16. **a)** A group of friends get together to make friendship bracelets. A package of embroidery floss can be shared equally among 3, 5, or 6 friends with no strands left over. What is the least number of strands the package can contain?

b) Suppose the package in part a could also be shared equally between 2 friends. Does this change your answer to part a? Why or why not?

17. A common multiple of two numbers is 64.

a) How could you find the two numbers?

b) Is there more than one possible answer?

c) If your answer to part b is yes, find as many pairs of numbers as you can.

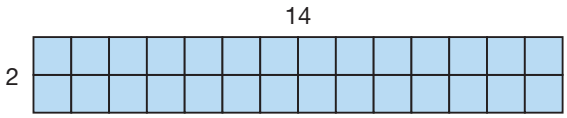
Reflect

Write your own problem you could solve using multiples.
Solve your problem.

LESSON 4

Prime and Composite Numbers

Numbers multiplied to form a product are factors of the product.



$2 \times 14 = 28$

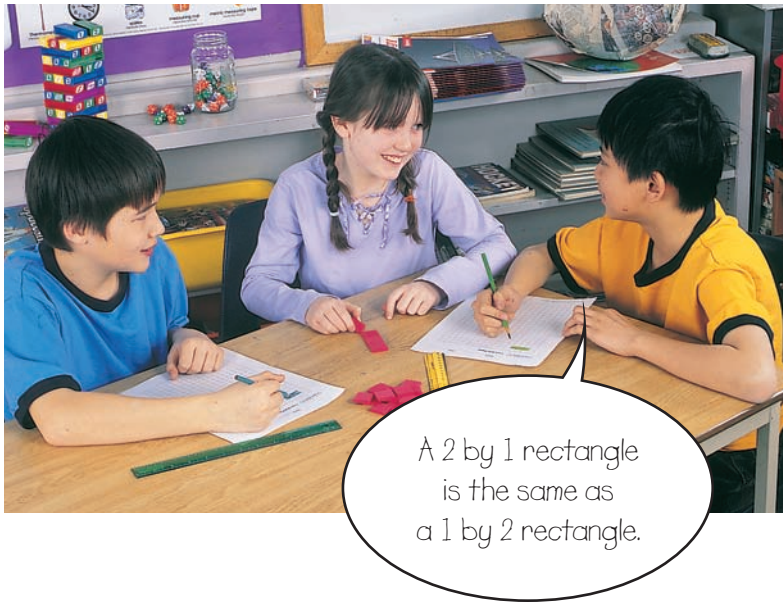
factor factor product

2 and 14 are factors of 28.
What are other factors of 28? How do you know?

Explore

You will need Colour Tiles or congruent squares and grid paper.

- Find all the different rectangles you can make using each number of tiles from 2 to 20.
  Draw each rectangle on grid paper.
  Write a multiplication sentence that describes the number of tiles in each rectangle.
- For which numbers of tiles could you make only 1 rectangle?
  For which numbers of tiles could you make 2 rectangles?
  3 rectangles?



Show *and* Share

Share your work with another group of students.
What are the factors of 2? Of 3?
What are the factors of 16? Of 20?
How could you find the factors of a number without making rectangles?

Connect

➤ Suppose you have 23 Colour Tiles.
You can make only 1 rectangle with all 23 tiles.

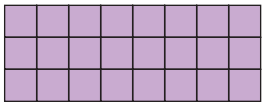
23 has 2 factors: 1 and 23
A number with exactly 2 factors,
1 and itself, is a **prime number**.
23 is a prime number.

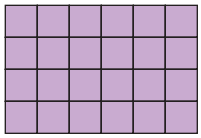
A prime number is a number greater than 1 that is divisible only by 1 and itself.

➤ Suppose you have 24 Colour Tiles.
You can make 4 different rectangles with 24 tiles.


$1 \times 24 = 24$


$2 \times 12 = 24$


$3 \times 8 = 24$


$4 \times 6 = 24$

24 has 8 factors: 1, 2, 3, 4, 6, 8, 12, and 24
The factors that are prime numbers are 2 and 3.

Here are 2 different strategies students used to find factors.

- Yao used multiplication facts to find all the factors of 40.
  She looked for whole numbers whose product is 40.

$1 \times 40 = 40$	1 and 40 are factors of 40.
$2 \times 20 = 40$	2 and 20 are factors of 40.
$4 \times 10 = 40$	4 and 10 are factors of 40.
$5 \times 8 = 40$	5 and 8 are factors of 40.

40 has 8 factors: 1, 2, 4, 5, 8, 10, 20, and 40
The factors that are prime numbers are 2 and 5.

- Maddie used arrays to find all the factors of 18.


$1 \times 18 = 18$


$2 \times 9 = 18$


$3 \times 6 = 18$

The factors of 18 are: 1, 2, 3, 6, 9, and 18
The factors that are prime numbers are 2 and 3.
Every number has at least 2 factors: 1 and the number itself
A number with more than 2 factors is a **composite number**.

Practice

You may use Colour Tiles or counters to model your solutions.

1. List all the factors of each number.

a) 6 **b)** 9 **c)** 25 **d)** 30 **e)** 12
f) 50 **g)** 28 **h)** 98 **i)** 20 **j)** 63

2. **a)** Name a prime number.
Explain how you know it is a prime number.
b) Name a composite number.
Explain how you know it is a composite number.

3. Which numbers below are factors of 80?
How do you know?

a) 2 **b)** 3 **c)** 4 **d)** 5
e) 6 **f)** 8 **g)** 9 **h)** 10

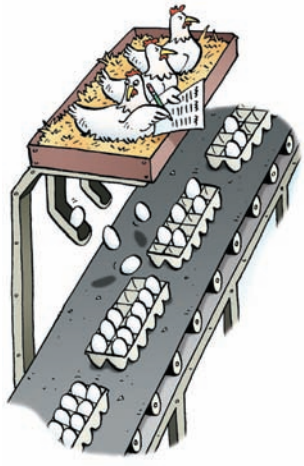
4. Which of the numbers 2, 3, 4, 5, 6, 8, 9, 12, 15, 17, and 19 are factors of:

a) 24? **b)** 38? **c)** 45? **d)** 51?

What strategy did you use to find out?

5. Eggs are packaged in cartons of 12.
Which of these numbers of eggs can be packaged in full cartons? How do you know?

a) 96 **b)** 56 **c)** 60 **d)** 74



6. Write 3 numbers between 30 and 50 that have:

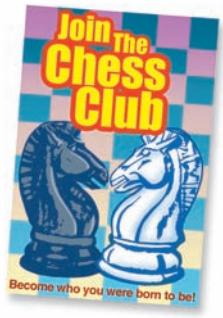
a) exactly 2 factors each **b)** more than 2 factors each

7. Write 3 numbers less than 100 that have exactly 4 factors each.

8. Sort these numbers as prime or composite.
How did you decide where to place each number?

59 93 97 87 73 45

9. Between 20 and 28 students signed up for the chess club.
The students could not be divided exactly into groups of 2, 3, 4, or 5.
How many students signed up for the chess club?
Show your work.

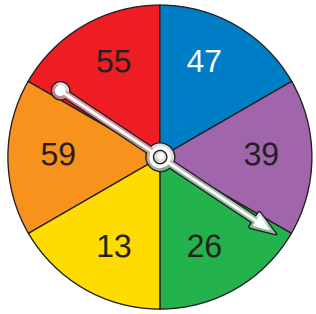


10. How many numbers between 70 and 80 are prime numbers?
What numbers are they?
Explain how you know they are prime numbers.

11. How many days in September have a prime number date?
How many have a composite number date?
Show how you know.

12. How can you tell that 32 and 95 are not prime numbers without finding their factors?

13. Brigitte and Stéphane play a game with this spinner.
Brigitte gets a point if the pointer lands on a prime number.
Stéphane gets a point if the pointer lands on a composite number.
The first person to get 20 points wins. Who is more likely to win?
How do you know?



14. A student said, "All prime numbers except for the number 2 are odd.
So, all odd numbers must be prime numbers."
Do you agree with the student? Explain.

15. Copy this Carroll diagram.

	Prime	Composite
Even		
Odd		

Sort the numbers from 2 to 30.

Reflect

Both 0 and 1 are neither prime nor composite.
Explain why.

LESSON 5

Investigating Factors

The factors of 6 are 1, 2, 3, and 6.
A number is *perfect* when all its factors, other than the number itself, add up to the number.
$1 + 2 + 3 = 6$
So, 6 is a perfect number.

Explore

➤ There are 50 people practising martial arts in Kinsman Park, Saskatoon.
Is 50 a perfect number? Explain how you know.

➤ How many students are in your class?
Is it a perfect number?
If not, how many more or fewer students would you need to make a perfect number? Show your work.

Kinsman Park, Saskatoon

Show *and* Share

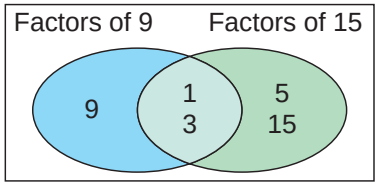
Share your work with another pair of students.
What strategies did you use to find the factors of each number?
How do you know you found all the factors?

Connect

You used factors to find perfect numbers.

➤ When we find the same factors for 2 numbers, we find **common factors**.

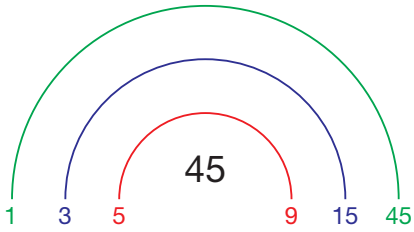
We can show the factors of 9 and 15 in a Venn diagram.
The factors of 9 are: 1, 3, 9
The factors of 15 are: 1, 3, 5, 15



The common factors of 9 and 15 are in the overlapping region.
The common factors of 9 and 15 are 1 and 3.

➤ Every composite number can be written as a product of its factors.

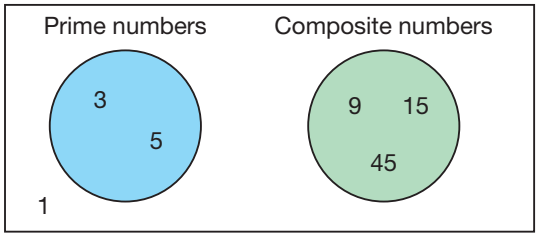
- We can use division facts to find all the factors of 45.
  For example, $45 \div 1 = 45$; $45 \div 3 = 15$; $45 \div 5 = 9$
  We can record the factors as a "rainbow."



If you are systematic, you are less likely to make errors.

There are no numbers between 5 and 9 that are factors of 45.
So, we know we have found all the factors.

The factors of 45 are: 1, 3, 5, 9, 15, 45
Some of the factors are prime numbers.
We can sort the factors:

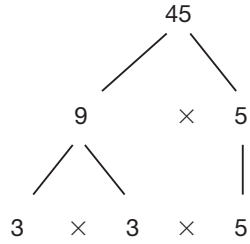


Here are two ways to find the factors of 45 that are prime.

- Draw a factor tree.

  Write 45 as the product of 2 factors.
  9 and 5 are factors of 45.
  9 is a composite number, so we can factor again.

  So, 3 and 5 are the factors of 45 that are prime numbers.



- Use repeated division by prime numbers.

  Begin by dividing 45 by the least prime number that is a factor: 3

  Divide by this prime number until it is no longer a factor.

  Continue to divide each quotient by a prime number until the quotient is 1.

  The factors of 45 that are prime numbers are 3 and 5.

$$3\overline{)45}\quad 15$$
$$3\overline{)15}\quad 5$$
$$5\overline{)5}\quad 1$$

Practice

1. Use a Venn diagram. Show the factors of 18 and 24.
   What are the common factors of 18 and 24?

2. Find the common factors of each pair of numbers.
   a) 15, 25 **b)** 16, 40 **c)** 18, 42 **d)** 35, 60

3. Find all the factors of each number.
   Record the factors as a "rainbow."
   a) 48 **b)** 50 **c)** 78 **d)** 62

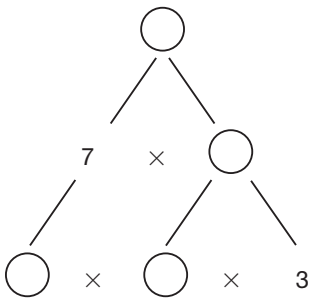
4. List all the factors of each number.
   How do you know you have found all the factors?
   Sort the factors into prime numbers and composite numbers.
   What do you notice?
   a) 34 **b)** 40 **c)** 72 **d)** 94

5. Draw a factor tree to find the factors of each number that are prime.
   a) 64 **b)** 85 **c)** 90 **d)** 76

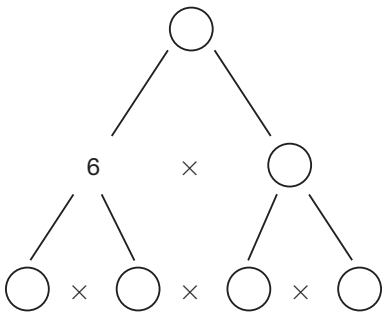
6. Use division to find the factors of each number that are prime.
   a) 18 **b)** 35 **c)** 36 **d)** 50

7. Use mental math to find the factors of each number that are prime.
   a) 15 **b)** 6 **c)** 21 **d)** 33

8. Copy and complete each factor tree in as many different ways as you can.
   a)



   b)



9. Patan uses a bead loom to make a bracelet.
She wants to use all 84 beads, and to put the beads in rows of equal length.
Patan also wants the number of beads in each row to be a factor of 84 that is a prime number.
How many beads could Patan put in each row?
Give as many answers as you can.
Explain how you found the numbers.

Bead Loom

10. Julia and Sandhu bought packages of granola bars.
Each package has the same number of bars.

a) Julia and Sandhu each had a total of 12 bars.
How many bars could there be in one package?

b) Suppose Julia had 24 bars and Sandhu had 18 bars.
How many bars could there be in one package?
Draw a picture to show your thinking.

11. Choose any 2-digit number.
Write clues to help a classmate guess your number.
One or more of your clues should be about factors.

12. **a)** Draw 2 different factor trees for each number.
i) 56 **ii)** 32 **iii)** 90 **iv)** 75

b) Why is it possible to draw 2 different factor trees for each number in part a?

c) Name 2 composite numbers for which you can draw only one factor tree.
Explain why this is so.

d) How many factor trees can you draw for the number 67? Explain.

13. Is your age a perfect number?
If it is not, when will your age be a perfect number?

14. A number is *almost perfect* when all its factors, other than the number itself, add up to one less than the number.
There are two numbers between 5 and 20 that are almost perfect.
Find these numbers.

Reflect

Which method for finding factors do you prefer?
Explain your choice.

The Factor Game

Play with a partner.
You will need one game board and 2 coloured markers.
The object of the game is to circle the factors of a number.
Decide who will be Player A and Player B.

- Player A circles a number on the game board and scores that number. Player B uses a different colour to circle all the factors of that number not already circled. She scores the sum of the numbers she circles.

  For example, suppose Player A circles 18.
  Player B circles 1, 2, 3, 6, and 9 (18 is already circled) to score $1 + 2 + 3 + 6 + 9 = 21$ points

- Player B circles a new number.
  Player A circles all the factors of that number not already circled. Record the scores.

- Continue playing.
  If a player chooses a number with no factors left to circle, the number is crossed out. The player loses her or his turn, and scores no points.

  For example, if player A circled 16, but 1, 2, 4, and 8 have already been circled, he would lose his turn and score no points.

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

- The game continues until all numbers have been circled or crossed out. The player with the higher score wins.

LESSON 6

Strategies Toolkit

Explore

Tarra has 10 clown fish and 15 snails.
She wants to place all of them in fish tanks so each tank has the same number of fish and the same number of snails.
What is the greatest number of tanks Tarra can set up?

You may use any materials to model your solution.
Record your solution.

Show *and* Share

Describe the strategy you used to solve the problem.

Connect

Twenty-four girls and 18 boys are forming teams.
All the children are on a team.
Teams must have equal numbers of girls and equal numbers of boys.
What is the greatest number of teams that can be formed?

What do you know?
- There are 24 girls and 18 boys.
- Boys and girls should be divided equally among the teams.

Think of a strategy to help you solve the problem.
- You can **make an organized list**.
- How many girls and how many boys are on each of 2 teams? 3 teams?

Strategies

- **Make a table.**
- **Solve a simpler problem.**
- **Guess and test.**
- **Make an organized list.**
- **Use a pattern.**

Lesson Focus | Interpret a problem and select an appropriate strategy.

Can you make 4 teams? 5 teams? 6 teams?
Explain.
What is the greatest possible number of teams?
How many girls and how many boys
will be on each team?

Check your work.
Did you find the greatest number of teams?
Does each team have the same number of girls
and the same number of boys?
How could you have used common factors
to solve this problem?

Practice

Choose one of the **Strategies**

1. Keshav is making party bags for his mystery party.
He has 40 notepads, 32 plastic magnifying glasses, and 16 fingerprinting kits.
Keshav wants to make as many party bags as possible.
He wants all the bags to be the same.
   a) How many party bags can Keshav make?
   b) How many notepads, magnifying glasses, and fingerprinting kits will be in each bag?
   How do you know?

2. Macie has 36 photos of Kakinga, her favourite gorilla at the Calgary Zoo.
She wants to arrange the photos in groups that have equal numbers of rows and columns.
How many different arrangements can Macie make?
Show your work.

Kakinga, Calgary Zoo

Reflect

Explain how an organized list can help you solve a problem.

LESSON 7

Order of Operations

Which operations would you use to find the answer to this question?
$18 - 6 \div 3 = ?$

Explore

To win a contest, Harry's dad must answer this skill-testing question:

$9 + 3 \times 6 - 4 =$ _____

- Find the answer in as many ways as you can.
- Record the strategy you use for each method.
- The correct answer is 23.
  Which strategy gives this answer?

Show *and* Share

Share your work with another student.
Discuss how to rewrite the question so the only possible answer is 23.

Connect

When you solve a problem that uses more than one operation, the answer depends on the order in which you perform the operations.

Evaluate the expression: $3 + 6 \times 4$
If you add first, you get: $9 \times 4 = 36$
If you multiply first, you get: $3 + 24 = 27$

To avoid getting two answers, there is a rule that multiplication is done before addition.

So, $3 + 6 \times 4 = 3 + 24$
$= 27$, which is the correct answer

An **expression** is a mathematical statement with numbers and operations. When we calculate the answer, we *evaluate* the expression.

We use brackets if we want certain operations carried out first. To make sure everyone gets the same answer when evaluating an expression, we use this order of operations:

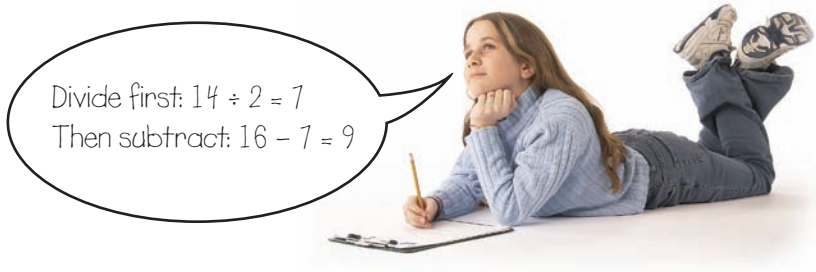
- Do the operations in brackets.
- Multiply and divide, in order, from left to right.
- Then add and subtract, in order, from left to right.

➤ Evaluate: $16 - 14 \div 2$

$16 - 14 \div 2$

$= 16 - 7$

$= 9$



➤ Evaluate: $18 - 10 + 6$

$18 - 10 + 6$

$= 8 + 6$

$= 14$

➤ Evaluate: $7 \times (4 + 8)$

$7 \times (4 + 8)$

$= 7 \times 12$

$= 84$

Some calculators follow the order of operations.
Others do not.
Check to see how your calculator works.

Practice

1. Evaluate each expression.
Use the order of operations.

a) $18 + 4 \times 2$ **b)** $25 - 12 \div 3$ **c)** $24 + 36 \div 9$
d) $12 - 8 - 4$ **e)** $50 - 7 \times 6$ **f)** $7 \times (2 + 9)$
g) $81 \div 9 - 6$ **h)** $25 \div (9 - 4)$ **i)** $13 - 6 + 8$
j) $(9 + 6) \div 3$ **k)** $19 + 56 \div 8$ **l)** $8 \times (12 - 5)$

2. Does your calculator follow the order of operations?
Press: 9 [+] 6 [×] 3 [=]
Explain how you know.

3. Bianca entered 52 [+] 8 [×] 2 [=] in her calculator.
She got the answer 120.
In what order did Bianca's calculator perform the operations?
How do you know?

4. Use a calculator to evaluate each expression.

a) $332 - 294 \div 49$ **b)** $209 \times 12 \div 4$
c) $312 \times 426 - 212 \times 158$ **d)** $2205 + 93 \div 3 - 1241$
e) $156 \times 283 + 215 \times 132$ **f)** $245 \times 138 \div (7 + 23)$
g) $(148 + 216) \times (351 - 173)$ **h)** $1258 + 341 \times 28 - 2357$

5. Use mental math to evaluate.

a) $20\,000 - 4000 \times 2$ **b)** $6 + 125 \div 25$
c) $(1000 + 6000) \times 3$ **d)** $60 \times 3 \div 9$
e) $5 \times (4 + 11)$ **f)** $50 + 50 \div 50$
g) $(50 + 50) \div 50$ **h)** $9 \times 10 - (30 + 30)$
i) $16 \div 2 \times 9$ **j)** $200 - 200 \div 20$

6. Use mental math to evaluate.

a) $4 \times 7 - 2 + 1$ **b)** $4 \times (7 - 2) + 1$
c) $4 \times 7 - (2 - 1)$ **d)** $4 \times (7 - 2 + 1)$
e) $(4 \times 7 - 2) + 1$ **f)** $4 \times 7 - (2 + 1)$

Which expressions give the greatest answer?
The least answer?

7. How many different answers can you get by inserting one pair of brackets in this expression?
$10 + 20 - 12 \div 2 \times 3$
Write each expression, then evaluate it.

8. Use the numbers 2, 3, and 4 and any operations or brackets.
Write an expression that equals each number below.
Try to do this more than one way.

a) 9 **b)** 10 **c)** 14 **d)** 20 **e)** 6

9. Alexi bought 5 T-shirts for $12 each and 3 pairs of socks for $2 a pair.
Which expression shows how much Alexi spent in dollars? How do you know?

a) $5 \times 12 \times 3 \times 2$

b) $5 \times 12 + 3 \times 2$

c) $(5 + 3) \times (12 + 2)$

10. Choose mental math, a calculator, or paper and pencil to evaluate.
For each question, how did you decide which method to use?

a) $238 - (2 \times 73)$ **b)** $47 \times (16 \times 18)$

c) $(36 + 14) \div 10$ **d)** $36 \times (48 \times 8)$

e) $60 \times (4 \div 2)$ **f)** $(200 + 50) \times (9 \div 3)$

11. Monsieur Lefèvre bought 2 boxes of fruit bars for his 3 children.
Each box has 6 fruit bars.
The children shared the fruit bars equally.
How many fruit bars did each child get?
Write an expression to show the order of operations you used.

12. Copy each number sentence.
Use brackets to make each number sentence true.

a) $36 \div 4 \times 3 = 3$

b) $20 \div 5 \times 2 + 3 = 5$

c) $10 - 4 \div 2 - 1 = 6$

d) $6 \times 2 + 8 \div 4 = 15$

Reflect

Why do we need rules for the order in which we perform operations?
Give examples to support your answer.

LESSON 8

What Is an Integer?

Temperature is measured in degrees Celsius (°C).
Water freezes at 0°C.

On a typical summer day in La Ronge, Saskatchewan, the temperature might be 24 degrees Celsius above zero.

A temperature greater than 0°C is positive.
We write: +24°C
We say: twenty-four degrees Celsius

On a typical winter day in La Ronge, the temperature might be 18 degrees Celsius below zero.

A temperature less than 0°C is negative.
We write: −18°C
We say: minus eighteen degrees Celsius

Explore

Use a positive or negative number to represent each situation.

- eight degrees above zero
- ten degrees below zero
- parking three levels below ground level
- twenty-three metres above sea level in Victoria, BC
- a loss of sixteen dollars
- taking four steps backward

Suppose you change the sign of each number.
What situation would each number now represent?

Butchart Gardens, Victoria

Show *and* Share

Compare your answers with those of another pair of students.
For each situation, how did you decide whether to use a positive number or a negative number?

Lesson Focus Use integers to describe quantities with size and direction.

Connect

Numbers such as +24 and −18 are **integers**.
The + sign in front of a number tells that it is a **positive integer**.
The − sign in front of a number tells that it is a **negative integer**.

➤ We can use coloured tiles to represent integers.

One yellow tile represents +1.

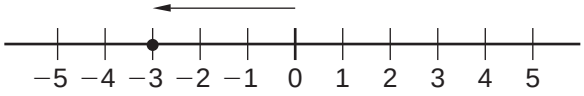
One red tile represents −1.

To model +6, we use 6 yellow tiles.

To model −5, we use 5 red tiles.

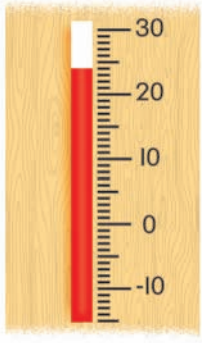
➤ We can show integers on a horizontal or vertical number line.

You have used horizontal number lines with whole numbers.
We extend the number line to the left of 0 to show *negative integers.*

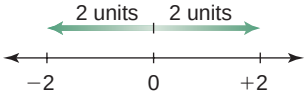


The arrow on the number line represents −3.
−3 is a negative integer. We say, "Negative 3."

A thermometer is a vertical number line.



➤ **Opposite integers** are the same distance from 0 but are on opposite sides of 0.
For example, +2 and −2 are opposite integers.
They are the same distance from 0 and are on opposite sides of 0.



+4 and −4 are also opposite integers, as are −21 and +21.

➤ We use integers to represent quantities that have both size and direction.

- Mark saved $25.
  This can be represented as $+\$25$, or $\$25$.
- A scuba diver swam to a depth of 50 m.
  This can be represented as -50 m.

If no sign is written, the integer is positive.

Practice

1. Write the integer modelled by each set of tiles.

a) 

b) 

c) 

d) 

2. Use yellow or red tiles to model each integer. Draw the tiles.

a) -6 **b)** $+8$ **c)** $+5$ **d)** -2

e) $+11$ **f)** -4 **g)** $+2$ **h)** -9

3. Mark these integers on a number line.
Tell how you knew where to place each integer.

a) $+1$ **b)** -5 **c)** -2 **d)** $+9$

4. Write the opposite of each integer.
Mark each pair of integers on a number line.
Describe any patterns you see.

a) $+3$ **b)** -1 **c)** -19 **d)** $+10$

5. Write an integer to represent each situation.

a) Sascha dug a hole 1 m deep.

b) Vincent deposited $50 in his bank account.

c) A plane flies at an altitude of 11 000 m.

d) A submarine travels at a depth of 400 m.

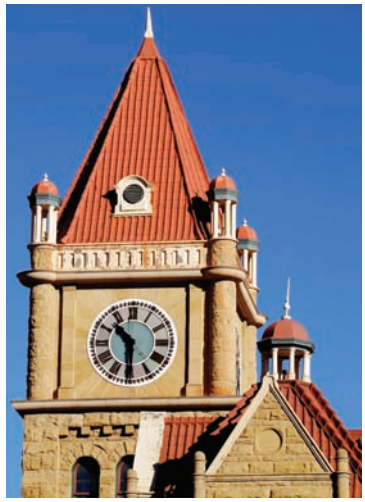
6. Use an integer to represent each situation.
Then use yellow or red tiles to model each integer. Draw the tiles.

a) 12°C below zero
b) 10 m above sea level
c) 9 s before take-off
d) a drop of $2 in the price of a movie ticket
e) a parking spot 5 levels below ground level

7. Describe a situation that could be represented by each integer.

a) 125 **b)** −22 **c)** −900
d) 42 000 **e)** 4

8. Describe two situations in which you might use negative and positive integers. Write integers for your situations.



Clock Tower, Calgary's Old City Hall

9. We measure time in hours.
Suppose 12 noon is represented by the integer 0.

a) Which integer represents 1 P.M. the same day?
b) Which integer represents 10 A.M. the same day?
c) Which integer represents 12 midnight the same day?
d) Which integer represents 10 P.M. the previous day?

Describe the strategy you used to find the integers.

10. Statistics Canada reported these data about Canada's population.

Years	Births	Deaths	Immigration	Emigration
1961–1966	2 249 000	731 000	539 000	280 000
1996–2001	1 705 000	1 089 000	1 217 000	376 000

a) Which numbers can be represented by positive integers? By negative integers? Explain your choices.
b) Choose one time period. Use a number line to explain the relationship between births and deaths.

Reflect

Suppose you read about a situation that can be described with integers.
What clues do you look for to help you decide whether to use a positive integer or a negative integer?
Use examples in your explanation.

LESSON 9

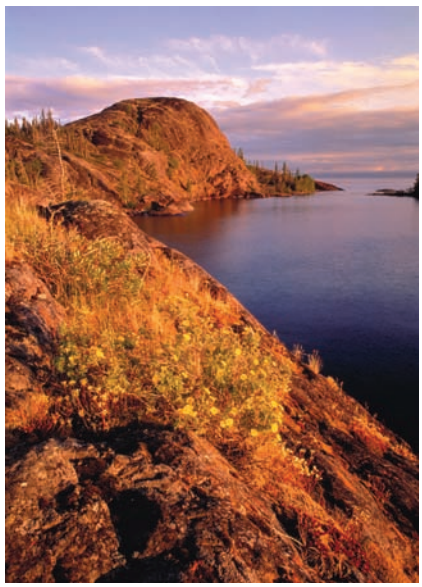
Comparing and Ordering Integers

Elevation is the height above or below sea level. Elevation influences climate and how people live. For example, crops will not grow at elevations above 5300 m.

Explore

You will need an atlas or Internet access.
Here are some examples of extreme elevations around the world.

Place	Elevation
Vinson Massif, Antarctica	4897 m above sea level
Dead Sea, Israel/Jordan	411 m below sea level
Bottom of Great Slave Lake, Canada	458 m below sea level
Mt. Nowshak, Afghanistan	7485 m above sea level
Challenger Deep, Pacific Ocean	10 924 m below sea level



Great Slave Lake, NWT

Find at least 4 more extreme elevations.
Two should be above sea level, and two should be below sea level.
At least one elevation should be in Canada.

Order *all* the elevations from least to greatest.

Show *and* Share

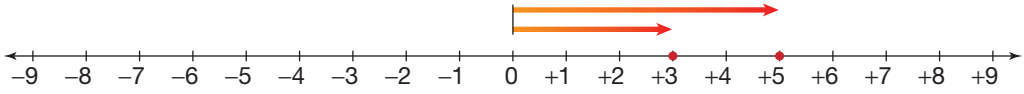
What strategies did you use to order the elevations? What other ways could you display these data to show the different elevations?

Lesson Focus | Use a number line to order and compare integers.

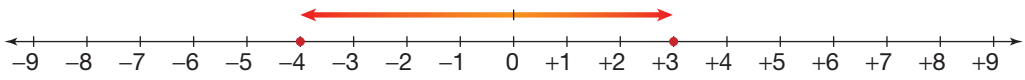
Connect

We can use a number line to order integers.

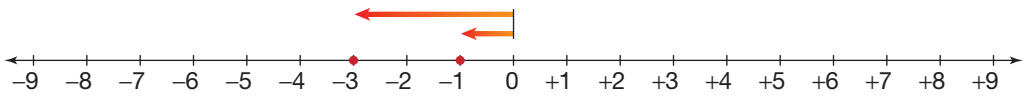
➤ We use the symbols > and < to show order.
The symbol points to the lesser number.



+5 is to the right of +3 on a number line.
+5 is greater than +3, so we write: $+5 > +3$
+3 is less than +5, so we write: $+3 < +5$



+3 is to the right of −4 on a number line.
+3 is greater than −4, so we write: $+3 > -4$
−4 is less than +3, so we write: $-4 < +3$



−3 is to the left of −1 on a number line.
−3 is less than −1, so we write: $-3 < -1$
−1 is greater than −3, so we write: $-1 > -3$

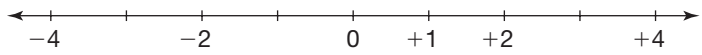
➤ To order the integers 0, +1, −2, +3, and −5,
draw a number line from −6 to +6.
Mark each integer on the number line.

The integers increase from left to right.
So, the integers from least to greatest are:
−5, −2, 0, +1, +3

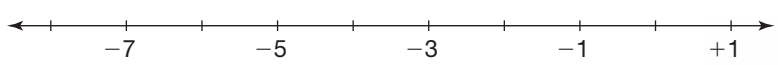
Practice

1. Copy each number line. Fill in the missing integers.

a)



b)

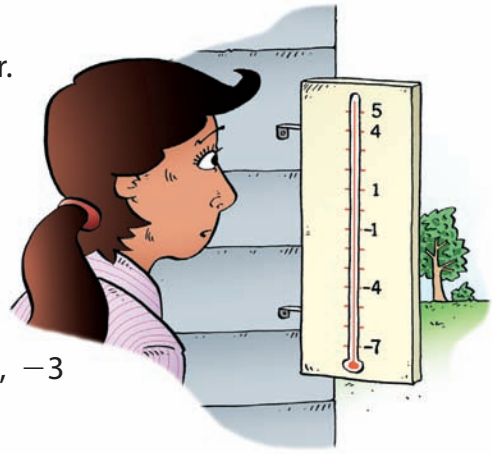


2. Six temperature markings are shown on the thermometer.

a) Which temperatures are greater than 0°C?

b) Which temperatures are less than 0°C?

c) Which temperatures are opposite integers? How do you know?



3. Which integer is greater? How did you find out?

a) +4, +3 **b)** +4, −3 **c)** −4, +3 **d)** −4, −3

4. Mark each set of integers on a number line.
Use the number line to order the integers from least to greatest.

a) +5, +13, +1 **b)** −3, −5, −4 **c)** +4, −2, +3

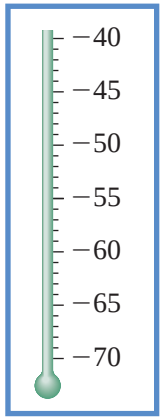
5. Use a number line. Order the integers in each set from greatest to least.

a) +4, +1, +8 **b)** −7, −5, −3 **c)** 0, +4, −4

6. This table shows the coldest temperatures ever recorded in 6 provinces and territories.

a) Draw a thermometer like the one shown. Mark each temperature on it.

Province/ Territory	Coldest Temperature (°C)
Alberta	−61
Manitoba	−53
Nova Scotia	−47
Nunavut	−64
Ontario	−58
Quebec	−54



Dog Sledding in Nunavut

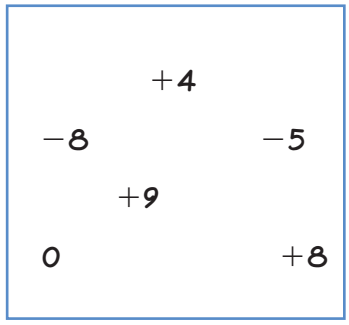
b) Order the temperatures in part a from least to greatest. How can you use your thermometer to do this?

7. Copy and complete by placing $<$, $>$, or $=$ between the integers.
Then, use a number line to verify your answer.

a) $+5 \square +10$ **b)** $-5 \square -10$ **c)** $+5 \square 5$ **d)** $-6 \square 0$

e) $-5 \square -4$ **f)** $10 \square -11$ **g)** $-8 \square -4$ **h)** $-8 \square -8$

8. Look at the integers in the box.



a) Which integers are:

i) greater than 0?

ii) between -3 and $+3$?

iii) greater than -10 and less than -5?

iv) less than $+1$?

b) What other questions can you ask about these integers?
Write down your questions and answer them.

9. Order the integers in each set from least to greatest.

a) $+5, -5, +4, +2, -2$ **b)** $-8, -12, +10, 0, -10$

c) $+41, -39, -41, -15, -25$ **d)** $+1, -1, +2, -2, +3$

10. Order the integers in each set from greatest to least.

a) $-7, +8, -9, +10, -11$ **b)** $-18, 16, -11, -4, +6$

c) $0, +1, +2, -1, -2$ **d)** $+14, -25, -30, +3, -10$

11. On January 16, 2008, these temperatures were recorded in Canada.

Place	Temperature	Place	Temperature
Lethbridge, AB	–16°C	Iqaluit, NU	–29°C
La Ronge, SK	–27°C	Dawson City, YT	–26°C
Hay River, NWT	–29°C	Prince George, BC	–6°C
Campbell River, BC	0°C	Ste. Rose du Lac, MB	–17°C

Which place was the warmest? The coldest? How did you find out?

12. **a)** Which of these integers are greater than -6? How do you know?
$-3, +2, -7, -5$

b) Which of these integers are less than -3? How do you know?
$+2, -11, +3, -2, -4$

13. You know that 8 is greater than 3. Explain why -8 is less than -3.

Reflect

When two integers have different signs, how can you tell which is greater?
When two integers have the same sign, how can you tell which is greater?

Unit 2 Show What You Know

LESSON

1

1. Write each number in standard form.

a) 3 billion 400 thousand 7 hundred

b) 20 000 000 + 3 000 000 + 60 000 + 4000 + 900 + 7

c) twenty-seven trillion fifty-seven million three hundred twenty-four thousand eighty-three

2. Write each number in expanded form.

a) 86 209 402 **b)** 23 854 265 001

2

3. Mrs. Wisely has $635 000 in the bank.
How much more money does she need before she can call herself a millionaire?
How did you decide which operation to use?

4. Top Tickets sells tickets for the Olympic Figure Skating Gala Exhibition, where all the medal-winning skaters perform. Use the table below.

Tickets Sold by Top Tickets		
Seating Level	**Price**	**Number Sold**
A	$525	126
B	$325	348
C	$175	1235

2006 Olympic Figure Skating Gala

a) How much money did Top Tickets take in?

b) Suppose Top Tickets wants to take in $700 000.
How much more money do they need to take in?

c) Suppose Top Tickets sold $284 725 worth of Level C tickets.
How many Level C tickets did they sell?

3

5. Which numbers below are multiples of 7?
How did you find out?
24 35 42 27 63 96 84

6. Find a common multiple of 4, 5, and 6.
Explain how you know the number you found is a common multiple.

LESSON

4

7. Tell if each number is prime or composite. How do you know?

a) 18 **b)** 21 **c)** 48 **d)** 37

8. Only one prime number is even.
Which number is it? How do you know it is a prime number?

5

9. List all the factors of each number.
Sort the factors into prime numbers and composite numbers.

a) 52 **b)** 28 **c)** 63 **d)** 76

10. Find the common factors of each pair of numbers.

a) 16, 32 **b)** 18, 27 **c)** 30, 75

11. Draw a factor tree to find the factors of each number that are prime.

a) 18 **b)** 48 **c)** 21 **d)** 75

7

12. Evaluate each expression.

a) $35 - 16 \div 4$ **b)** $8 \times (6 + 4)$ **c)** $86 - 9 \times 9$

13. Evaluate each expression.

a) $16\,974 - (18 \times 45)$ **b)** $8537 + 4825 \div 25$

8

14. Draw a number line. Mark each integer on the line.
How did you know where to place each integer?
$+3,\ -5,\ +1,\ -2,\ 0$

15. Use an integer to represent each situation.

a) Sandha skated backward 100 m.

b) Karl earned $140 mowing lawns.

c) The temperature in Alida, SK, was 12°C below zero.

d) The elevator went up 7 floors.

9

16. Use a number line.
Order the integers in each set from least to greatest.

a) $+4,\ -3,\ -2,\ +1,\ -4$

b) $+8,\ +5,\ 0,\ -5,\ -17$

c) $+10,\ -9,\ +8,\ -7,\ +6$

UNIT 2

Learning Goals

- ☑ use place value to represent whole numbers greater than one million
- ☑ solve problems involving large numbers, using technology
- ☑ determine multiples and factors of numbers less than 100
- ☑ solve problems involving multiples
- ☑ identify composite and prime numbers
- ☑ apply the order of operations to solve multi-step problems, with or without technology
- ☑ demonstrate an understanding of integers

Unit Problem

At the Apiary

Honeybees have 4 wings that beat about 11 400 times per minute.

The ideal temperature of a hive is 32°C. In winter, when the temperature can be −20°C, honeybees beat their wings to generate heat to keep the queen and her hive from freezing.

A honeybee flies an average of 22 km each hour.

A honeybee visits about 4400 flowers to gather enough nectar to make 10 g of honey.

During her busy season, the queen bee lays about 1500 eggs in 24 h.

In 2000, there were 603 828 hives in Canada. The average number of hives per beekeeper was 61, and the average yield of honey per hive was 52 kg.

Check List

Your work should show
- ☑ that you can choose the correct operation
- ☑ how you calculated and checked your solutions
- ☑ an interesting story problem involving numbers
- ☑ clear explanations of your solutions and strategies

Solve questions 1 to 3.
Use a calculator when you need to.
Check your solutions. Show all your work.

1. During her busy season, about how many eggs does the queen bee lay each hour? Each minute?

2. Each day, the queen bee eats 80 times her mass in food.
Suppose a cat needed to eat 80 times its mass each day.
How many kilograms of food would a cat eat each day? Each month?

3. The typical Canadian eats about 880 g of honey each year.
Millicent is 12 years old.
She estimates she has eaten about 11 kg of honey in her lifetime.
Is Millicent a typical Canadian honey eater? Explain.

4. Use some of the honeybee data on page 84 or use data you can find about honeybees.
Write a story problem.
Solve the problem.
Describe your strategy.

Reflect on Your Learning

Write about some of the things you have learned about numbers in this unit.